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Abstract

This paper proposes a model in which identical sellers of a homogenous product compete in both prices and price frames (i.e., ways to present price information). Frame choices affect the comparability of price offers, and may cause consumer confusion and lower price sensitivity. In equilibrium, firms randomize their frame choices to obfuscate price comparisons and sustain positive profits. The nature of equilibrium depends on whether frame differentiation or frame complexity is more confusing. Moreover, an increase in the number of competitors induces firms to rely more on frame complexity and this may boost industry profits and lower consumer surplus.

Keywords: bounded rationality, framing, oligopoly markets, frame dispersion, price dispersion
JEL classification: D03, D43, L13

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1 Introduction

Sellers often use various ways to convey price information to consumers. Retailers use different discount methods to promote their products, such as direct price reductions, percentage discounts, volume discounts, or vouchers.[1] Some restaurants, hotels, and online booksellers offer a single price, while others divide the price by quoting table service, breakfast, internet access, parking, or shipping fees separately. Airlines and travel agencies charge card payment fees in different ways. For instance, Wizz charges a flat £4 per person, while Virgin Atlantic charges 1.3% of the total booking.[2] Retailers offer store cards with diverse terms such as "10% off first shop if opened online or 10% off for the first week if opened in store", "500 bonus points on first order", or "£5 voucher after first purchase". Financial product prices are also often framed distinctively: mortgage arrangement fees might be rolled in the interest rate or not; some loans may specify the monthly interest rate, while others the annual interest rate. In some cases (e.g., supermarket promotions), sellers also change their price presentation formats over time.

Strategic choice of price presentation formats or, simply, price framing has received relatively little attention in the economics literature in spite of its prevalence. If firms use different price frames to compete better for consumers, industry-specific pricing schemes whose terms facilitate comparisons should emerge. But, persistent variation in price frames in some markets is more likely to confuse consumers and harm competition. In recent years, potentially confusing pricing has attracted public's and policy makers' attention. The main concern is that sellers might deliberately use price framing to obfuscate price comparisons and soften price competition. For example, the inconsistent use of unit prices (e.g., price per unit vs. price per weight) in British grocery stores has attracted criticism from consumer watchdogs. In utility markets, complex tariff structures may make it difficult for consumers to understand what type of deal they are on and how to reduce energy use and costs.[3]

This paper proposes a model where sellers of a homogeneous product can choose both price frames and prices. We assume that consumers might be confused by price framing and so fail to identify the best available deal in the market. Specifically, firms can choose either a simple frame or a potentially complex frame, and consumers may get confused when firms adopt different frames or when they adopt a common but complex frame.[4] If consumers get confused, they choose one product randomly; otherwise, they behave rationally. We show that in equilibrium

[1] For example, to buy a 50 ml whitening toothpaste in a grocery store one can choose a Macleans sold at £2.31 with a "buy one get one free" offer or an Aquafresh which "was £1.93 now is £1.28 saves 65p".

[2] See "Calls for airline charges clean-up" on BBC News on July 17, 2009 (*http://news.bbc.co.uk)*.

[3] UK's gas and energy market regulator, Ofgem has started to address complex tariffs (see its Factsheet 107 on "Simpler energy tariffs" from October 14, 2011.) See also the articles "Supermarkets 'confuse' consumers with product pricing" in *The Guardian* (November 17, 2011) and "Customers confused by energy tariffs" at *http://www.which.co.uk/news* (May 7, 2009).

[4] Psychology research has recognized framing effects in decision making (see Tversky and Kahneman, 1981). Individuals' responses to essentially the same decision problem may differ systematically if the problem is framed differently. Here, we focus on frames as price presentation modes and on their ability to cause confusion in price comparisons.

firms adopt mixed strategies that randomize on both price frames and prices, and make strictly positive profits in an otherwise homogeneous product market. Moreover, as the number of firms increases, it becomes more difficult to obfuscate price comparisons by adopting different frames, and firms use complex price frames more often. As a result, more competition might actually boost profits and harm consumers. Our model suggests that in the presence of price framing, a standard competition policy approach may have undesired effects on consumer welfare.

Marketing research provides evidence that consumers have difficulties in comparing prices that are presented differently or prices that are complicated (see, e.g., Estelami, 1997, Morwitz et al., 1998, and Thomas and Morwitz, 2009). Economics experiments (see, e.g., Kalayci and Potters, 2011, and Kalayci, 2011) show that increasing the number of product attributes or price scheme dimensions can create confusion and lead to suboptimal consumer choices. We explore two sources of consumer confusion due to price presentation: frame differentiation (when firms adopt different frames) and frame complexity (when firms use a common but complex frame).

Consider, for instance, the following two frames: "price per unit" and "price per kilogram".[5] In this case, comparing two prices in the same frame is easy, but comparing a price per unit with a price per kilogram might be difficult for some consumers. Here, frame differentiation is the confusion source. Other examples of incompatible price formats are price incl. VAT vs. price excl. VAT, flat card payment fees vs. percentage ones, and monthly interest rate vs. annual interest rate quotations on loans.

Now consider the frames "price incl. shipping fee" and "price plus shipping fee". Ranking all-inclusive prices is easy and, as before, comparing prices in different frames might be difficult. However, in this case comparing prices that quote separately the shipping fees may also be confusing if the fees vary across sellers. Here, frame complexity arising from the use of involved formats (two-dimensional prices) is also a source of consumer confusion. This is true in other settings (e.g., in financial services or utility markets) where some frames are involved multi-part tariffs.[6] For instance, mortgage deals with the service fee quoted separately are usually harder to compare than deals with the service fees rolled in the interest rate. When both sources of confusion coexist, it is not obvious which of them is more likely to confuse consumers. The answer depends on the microfoundations of confusion, which will be discussed in the modelling section. Hence, we consider both possibilities in our analysis.

Our study predicts both frame and price dispersion in the presence of price framing. When the market is relatively transparent (for instance, when both firms use a simple frame), a firm has an incentive to create more confusion through its price frame choice and take advantage of confused consumers. But when the market is already confusing enough (for instance, when firms use different frames), a firm has an incentive to reduce confusion by choosing the same frame as its rival and then undercut it. Due to this conflict, firms randomize on price frames,

[5] These price formats are commonly used, for example, in grocery stores for varieties of the same fruit or vegetable.

[6] The white paper on the integration of EU mortgage credit markets (2007) states that "even if consumers do have the relevant information [to make a choice] they do not necessarily understand it".

which generates both price aware buyers (who compare prices perfectly) and confused buyers (who shop at random). As a result, firms will also randomize on prices in equilibrium. This prediction is consistent with casual observations in many markets. Grocery stores and online retailers, for instance, use different price frames and also change their price formats over time. Moreover, in these markets there is substantial price dispersion.

The nature of equilibrium depends on which source of confusion leads to more confused consumers. If frame differentiation is the dominant confusion source, there is no clear ranking (on average) among prices associated with different frames. For example, in the case of price per unit vs. price per kilogram, there should be no significant price differences on average across different price formats. By contrast, in markets where frame complexity is the dominant source of confusion, the more complex frame is associated with higher prices. Woodward (2003) and Woodward and Hall (2010) provide evidence that, in the mortgage market, the deals with the arrangement fees rolled in the interest rate are on average better than the deals with separate fees.[7] The intuition behind these results is best illustrated in duopoly. When a firm shifts from a simple frame to a complex frame, more consumers get confused regardless of rival's choice if frame complexity is the dominant confusion source. Then, firms charge higher prices when they use the complex frame. By contrast, if frame differentiation is the dominant confusion source, when a firm shifts from a simple frame to a complex one, more or fewer consumers get confused depending on rival's frame choice. This implies that there is no monotonic price-frame relationship.

In our setting, deliberately choosing different or complex price frames is a way to make price comparisons difficult that ultimately allows suppliers of otherwise homogeneous products to obtain positive profits. We show that a decrease in concentration weakens firms' ability to frame differentiate and makes them overload on frame complexity. In particular, in fragmented oligopolies, firms tend to use complex price frames almost surely and industry profits are bounded away from zero regardless of the number of competitors. A decrease in concentration has a positive effect on consumer welfare (pressure down on prices), but also a negative one (higher market complexity and less competitive pressure). So, in the presence of price framing, when the latter effect dominates, an increase in the number of firms boosts industry profits and harms consumers.[8] Hortaçsu and Syverson (2004) provide evidence that in the S&P index fund market where multi-dimensional fee schemes prevail, a decrease in concentration between 1995-99 indeed triggered an increase in the average price.

Recent economics research investigates price complexity and firms' intentional attempts to degrade the quality of information to the consumers. Ellison and Ellison (2009) find empirical

[7] In this paper, consumers who lack the ability to compare prices are also assumed to lack the ability to understand the equilibrium relationship between frames and prices. This may be the case when consumers do not have the chance to learn the market equilibrium (e.g., because they do not buy the product frequently).

[8] Lower concentration may lead to higher prices in other price dispersion models (e.g., Rosenthal, 1980, and Janssen and Moraga-González, 2004) where competition makes firms exploit uninformed consumers, rather than fight for shoppers. However, our result stems from a novel effect of lower concentration on firms' framing incentives and the overall market complexity.

evidence on obfuscation strategies in online markets where retailers deliberately create more confusing websites to make it harder for the consumers to figure out the total price. Carlin (2009) and Ellison and Wolitzky (2008) address this issue in the information search framework where each firm chooses both a price and a price complexity level. They argue that if it is more costly for consumers to assess complex prices, each firm will individually increase price complexity to reduce consumers' incentives to gather information and weaken price competition.[9] Our model also considers price complexity, but it incorporates the effect of price frame differentiation and regards it as an important source of market complexity. In particular, in our model whether a firm's frame choice can soften price competition also depends on rivals' frame choices. This strategic dependence induces firms to randomize on frames. So our model predicts that firms tend to adopt different price frames or change their price frames over time.[10]

In a closely related paper, Piccione and Spiegler (2012) also examine frame-price competition. They focus on a duopoly model with a more general frame structure, and mainly examine the relation between equilibrium properties and the frame structure. Our duopoly example can be regarded as a special case of their model. However, we develop an oligopoly model to analyze the impact of greater competition on firms' strategies and market outcomes in the presence of price framing. In addition, our analysis explores the interaction between frame differentiation and frame complexity as sources of consumer confusion.

Our paper is also related to the recent literature on how shrouding a price component or making it less salient affects consumers' price perception and market outcomes. Ellison (2005) studies an add-on pricing model in which consumers have imperfect information about the prices of add-ons and need to pay a search cost to find them out. He shows that if the consumers who have a low valuation for the add-ons are also more price sensitive, then the existence of add-ons can soften price competition and sustain positive profits. Gabaix and Laibson (2006) endogenize shrouded add-on price information by introducing boundedly rational consumers who ignore the add-on price if it is not advertised. (See Hossain and Morgan (2006), Chetty, Looney, and Kroft (2009), and Brown, Hossain, and Morgan (2010) for related empirical work.) Our model does not explore hidden price information, but while all price information is available, framing affects choice due to consumers' limited cognitive abilities.

Finally, our study contributes to the growing literature on bounded rationality and industrial organization (see the survey by Ellison, 2006, and Spiegler, 2011). In our model, the inability of

[9] In a sequential search model, Ellison and Wolitzky (2008) consider a convex search cost function. If a firm makes its price information more costly to process, consumers are less likely to search further. In an all-or-nothing search model, Carlin (2009) assumes that if a firm increases its price complexity, consumers regard the entire market as being more complex. Then, as information gathering is more costly they are more likely to remain uninformed and shop randomly. See also Wilson (2010) for a two-stage duopoly model of obfuscation with asymmetric equilibrium price complexity levels.

[10] The price dispersion literature (see Baye et al., 2006 for a survey) associates mixed-strategy equilibria with both cross-sectional and intertemporal variations. Some of our examples relate to cross-sectional frame dispersion (e.g., the case of online bookstores), while others could be linked to intertemporal frame variation (e.g., the case of retail promotions).

boundedly rational consumers to compare framed prices leads to equilibrium frame dispersion. Our study is also related to the literature on consumer search and price dispersion. But, we focus on how firms may confuse consumers by mixing their frame choices, and in our model price dispersion is a by-product of frame dispersion.

2 The Duopoly Model

This section illustrates in a duopoly example the coexistence of frame and price dispersion and how the relative importance of frame differentiation and frame complexity affects the nature of equilibrium.

2.1 Model

Consider a market for a homogeneous product with two sellers, firms 1 and 2. The constant marginal cost of production is normalized to zero. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay at most 1. There are two possible price presentation modes, referred to as frames A and B. Frame A is a simple frame (e.g., a single all-inclusive price), and frame B is a different and potentially more complex frame (e.g., a basic price plus a shipping fee). Note that both frames may be simple (e.g., A is price per unit and B is pice per kilogram). Each firm can choose just one of the two frames, and they simultaneously and noncooperatively choose frames and prices p_1 and p_2. The timing reflects the fact that in many cases both price frames and prices can be changed relatively easily. If changing price frames takes longer than adjusting prices, a two-stage game where firms first commit to frames and then compete in prices will be more suitable. We discuss a sequential version of the model in the end of this section.

Price framing affects consumer choice as follows. If both firms choose the simple frame A, all consumers can perfectly compare the two prices and buy the cheaper product with a positive net surplus. Formally, in this case, firm i's demand is

$$q_i(p_i, p_j) = \begin{cases} 1, & \text{if } p_i < p_j \text{ and } p_i \leq 1 \\ 1/2, & \text{if } p_i = p_j \leq 1 \\ 0, & \text{if } p_i > p_j \text{ or } p_i > 1 \end{cases} \qquad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j. \tag{1}$$

If the two firms adopt different frames, a fraction $\alpha_1 > 0$ of consumers get confused and are unable to compare the two prices. The remaining $1-\alpha_1$ fraction of consumers can still accurately compare prices. This duopoly example, for simplicity, assumes that confused consumers shop at random: half of them buy from firm 1 and the other half buy from firm 2. In general, consumers might favor the firm adopting the simpler frame whenever they get confused between different frames. The oligopoly model in section 3 allows for such preferences.

If both firms choose frame B, a fraction $\alpha_2 \geq 0$ of consumers get confused and shop randomly. Note that $\alpha_2 = 0$ if frame B is also a simple (but different) frame. Table 1 shows the fraction

of confused consumers for all the frame profiles, where z_i is the frame chosen by firm i and z_j is the frame chosen by firm j.

Table 1: Confused consumers

$z_i \setminus z_j$	A	B
A	$\alpha_0 = 0$	α_1
B	α_1	α_2

We assume that nobody is confused if both firms use A for expositional reasons. The main results hold qualitatively if a fraction of consumers also get confused in this case, provided that $\alpha_0 \leq \alpha_2$ and $\alpha_0 \neq \alpha_1$.

Then, firm i's profit is

$$\pi_i(p_i, p_j, z_i, z_j) = p_i \cdot [\frac{1}{2}\alpha_{z_i,z_j} + q_i(p_i, p_j)(1 - \alpha_{z_i,z_j})] \ ,$$

where α_{z_i,z_j} is presented in Table 1 and $q_i(p_i, p_j)$ is given by (1).

In our model, confused consumers do not pay more than their reservation price equal to 1. Arguably, if price framing prevents a consumer from comparing competing offers, it may also prevent her from accurately comparing framed prices and her willingness to pay. In this case, one way to justify our assumption is that consumers can figure out at checkout (or after purchase) if a product's price exceeds their valuation and can decline to buy it (or return it). Given such ex-post participation constraint, firms have no incentive to charge prices above 1.[11] In addition, confused consumers are assumed to be unable to understand the relationship between price frames and prices. For example, even if a particular frame is always associated with higher prices, confused consumers are unable to infer prices from the price frame. This may be the case if consumers who lack the ability to compare prices are also unable to understand the market equilibrium. We revisit this issue in Section 4.

Our model explores two sources of consumer confusion: frame differentiation (i.e., prices are presented in incompatible formats) and frame complexity (i.e., prices are presented in a common involved format). If $\alpha_2 = 0$ (i.e., if frame B is also a simple frame), frame differentiation is the sole source of consumer confusion and it is captured by α_1. If $\alpha_2 > 0$, frame complexity is also a source of consumer confusion. When consumers face the frame profile (B, B), a fraction α_2 of consumers get confused solely because of frame complexity. When consumers face the frame profile (A, B), a fraction α_1 of consumers get confused due to frame incompatibility. (In the latter case, frame incompatibility might also stem from the fact that the profile involves a complex price and so both sources of confusion may be conceptually related.) The ranking

[11] Carlin (2009) and Piccione and Spiegler (2012) make the same assumption for tractability. Alternatively, suppose confused consumers pay up to $v > 1$. In the tractable case with $\alpha_2 = 0$, it can be shown that if $v < 2$, (i) there is no pure-strategy equilibrium, and (ii) the symmetric mixed-strategy equilibrium takes the same form as in our current setting except that the price distribution has a mass point on v (i.e., the new support is $[p_0, 1] \cup \{v\}$). (If $v \geq 2$, firms have incentives to just exploit confused consumers: they adopt different frames and charge a price $p = v$.)

of α_1 and α_2 reflects the relative importance of frame differentiation and frame complexity as sources of consumer confusion.

The relative role of the two confusion sources and their relevance in the marketplace stem from the microfoundations of consumer confusion. We present below two possible interpretations.

Frame differentiation dominates frame complexity ($\alpha_1 > \alpha_2$). When consumers face a simple frame A and a complex frame B, to compare the two offers they need to convert the price in frame B into a single all-inclusive price. Imagine that due to differences in numeracy skills, some consumers are able to make a correct conversion, while others are not. We assume that those who are unable to convert get confused and end up choosing randomly. When consumers face two offers in frame B, those who are able to convert B into A should still be able to compare. Moreover, those with poor numeracy skills may now benefit from format similarity. For example, if frame B is a two dimensional price and one offer dominates the other in both dimensions, then even those who are unable to convert will make the right choice. That is, similarity between the price formats may mitigate the confusion caused by frame complexity.[12] This is obvious, for example, when B is "price plus VAT" and the same tax rate applies. In this example, frame similarity rules out confusion (and B can be regarded as a simple frame).

Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$). Consumers might be able to convert a price presented in frame B into a simple price in frame A, but this requires costly information processing and consumers may decide whether or not to make the conversion. When they give up making the conversion, they end up confused. If confusion stems from this conversion cost, a consumer is more likely to give up the effort when she compares two complex prices than when she compares one complex price with a simple one. Then, the frame profile (B, B) leads to more confused consumers than the profile (A, B).

We use a reduced-form approach and do not explicitly model the comparison procedures that may lead to confusion. In reality, there may be several confusion mechanisms so that both cases of $\alpha_1 > \alpha_2$ and $\alpha_2 > \alpha_1$ are worth exploring.

Finally, in our setting confused consumers' choices are assumed to be totally independent of firms' prices. This is a tractable way to capture the idea that confusion in price comparisons reduces consumers' price sensitivity and weakens price competition. An alternative (but less tractable) model might assume that price framing leads to *noisy* price comparisons. Suppose firm i charges a price p_i. If it uses the simple frame A, consumers will understand its price perfectly. In contrast, if it uses frame B, consumers will perceive its price as $p_i + \varepsilon_i$, where ε_i is a random variable that captures possible misperceptions. Then, for example, if firm i adopts

[12] Even if there is no clear dominance relationship between offers, frame similarity may still facilitate comparison of prices framed in B. Take for example two offers in frame B: (1) £32.78 plus £4.75 shipping, and (2) £32.97 plus £4.32 shipping. When a consumer compares them, she may assess different components separately. The base price in (2) is about 20p higher than in (1), but the shipping fee in (2) is about 40p cheaper than in (1), so (2) is a better deal than (1). However, if the consumer needs to compare, say, (1) with a single price £37.25, it seems plausible that she has to convert (1) into an all-inclusive price first, which is more demanding in calculation and so it may block the comparison.

the relatively complex frame B and firm j adopts frame A, consumers perceive their prices as $p_i+\varepsilon_i$ and p_j, respectively. As a result, demand becomes less elastic compared to the case where both firms use frame A. (This is reflected by $\alpha_1 > 0$ in our current setting.) But consumers' choices still depend somewhat on the relative price $p_i - p_j$. We discuss this alternative model in Section 4 and relate the relative ranking of α_1 and α_2 to the correlations between the errors.

2.2 Analysis

Let us now characterize the duopoly equilibrium.[13] We first show that there is no pure-strategy framing equilibrium, and then we prove the existence and uniqueness of a symmetric mixed-strategy equilibrium. All proofs missing from the text are relegated to the Appendix A.

Lemma 1 *If $\alpha_1 \neq \alpha_2$, there is no equilibrium where both firms choose deterministic price frames.*

Proof. (a) Suppose both firms choose frame A for sure. Then, the unique candidate equilibrium entails marginal-cost pricing and zero profit. But, if firm i unilaterally deviates to frame B and a positive price (no greater than one), it makes a positive profit. A contradiction.
(b) Suppose both firms choose frame B for sure. For clarity, consider two cases. (b1) If $\alpha_2 = 1$ (and so $\alpha_1 < \alpha_2$), at the unique candidate equilibrium $p_i = 1$ and $\pi_i = 1/2$ for all i. But, if firm i unilaterally deviates to frame A and price $p_i = 1 - \varepsilon$, it earns $(1-\varepsilon)\left[\alpha_1/2 + (1-\alpha_1)\right] > 1/2$ for ε small enough. (b2) If $\alpha_2 < 1$, the unique candidate equilibrium dictates mixed strategy pricing according to a cdf on $[p_0, 1]$ as in Varian (1980), and each firm's expected profit is $\alpha_2/2 = p_0\left(1-\alpha_2/2\right)$.[14] If $\alpha_1 > \alpha_2$, firm i can make a higher profit $\alpha_1/2 > \alpha_2/2$ by deviating to frame A and price $p_i = 1$. If $\alpha_1 < \alpha_2$, firm i can make a higher profit $p_0\left(1-\alpha_1/2\right) > p_0\left(1-\alpha_2/2\right)$ by deviating to frame A and price $p_i = p_0$. Both (b1) and (b2) lead to a contradiction.
(c) Suppose firm i chooses frame A and firm j chooses B. Again consider two cases. (c1) If $\alpha_1 = 1$, the unique candidate equilibrium entails $p_i = 1$ and $\pi_i = 1/2$ for all i. But, then, firm j is better off deviating to frame A and $p_j = 1 - \varepsilon$, in which case its profit is $1 - \varepsilon > 1/2$ for any $\varepsilon < 1/2$. (c2) If $\alpha_1 < 1$, then the unique candidate equilibrium is again of Varian type and dictates mixed strategy pricing according to a cdf on $[p_0, 1]$, with each firm earning $\alpha_1/2 = p_0\left(1-\alpha_1/2\right)$. But if firm j deviates to frame A and price $p_j = p_0$, it makes a higher profit p_0. Both (c1) and (c2) lead to a contradiction.[15] ■

If both firms use the same simple frame (that is, A or, for $\alpha_2 = 0$, also B), they compete *à la* Bertrand and make zero profits. A unilateral deviation to the other frame yields positive

[13] Our duopoly example can be regarded as a reduced-form model of the bi-symmetric graph case in Piccione and Spiegler (2012). All their results apply to our model, but in our setting it is subtler to exclude the possibility of firms adopting deterministic frames. In their model, consumers are always able to perfectly compare prices in the same frame (i.e., frame differentiation is the only confusion source), so it is easy to see that firms never adopt deterministic frames.

[14] See Baye et al. (1992) for the uniqueness proof in the two-firm case.

[15] Although parts (a) and (c) used the fact that consumers can compare prices perfectly when both firms use frame A, our result still holds even if $\alpha_0 > 0$ provided that $\alpha_0 \neq \alpha_1$ (the logic in (b) applies).

profits as some consumers are confused by "frame differentiation" and shop at random. For $\alpha_2 > 0$, Lemma 1 also shows that in equilibrium, the firms cannot rely on only one confusion source. Otherwise, a firm using frame B has a unilateral incentive to deviate to the simpler frame A to attract price aware consumers. But, if $\alpha_1 = \alpha_2 > 0$, there is an equilibrium with both firms using frame B, as a unilateral deviation to frame A does not change the composition of consumers in the market.

In continuation, we focus on the general case with $\alpha_1 \neq \alpha_2$. By Lemma 1, in any candidate equilibrium at least one firm mixes its frame choice. Therefore, there is a positive probability that firms have bases of fully aware consumers, *and* also a positive probability that they have bases of confused consumers who cannot compare prices at all. The conflict between the incentives to fully exploit confused consumers and to vigorously compete for the aware ones leads to the absence of pure strategy pricing equilibria. The proof of the following result is standard and therefore omitted.

Lemma 2 *If $\alpha_1 \neq \alpha_2$, there is no equilibrium where both firms charge deterministic prices.*

Lemmas 1 and 2 show that any duopoly equilibria must exhibit dispersion in *both* price frames and prices. Let us now focus on the *symmetric mixed-strategy equilibrium* (λ, F_A, F_B) where each firm assigns probability $\lambda \in (0,1)$ to frame A and $1-\lambda$ to frame B and, when a firm uses frame $z \in \{A, B\}$, it chooses its price randomly according to a cdf F_z that is strictly increasing on its connected support $S_z = [p_0^z, p_1^z]$. We first show that F_z is continuous (except when $\alpha_2 = 1$).

Lemma 3 *In the symmetric mixed-strategy equilibrium (λ, F_A, F_B), the price distribution associated with frame A (F_A) is always atomless, and the one associated with frame B (F_B) is atomless whenever $\alpha_2 < 1$.*

Denote by

$$x_z(p) \equiv 1 - F_z(p)$$

the probability that a firm using frame z charges a price higher than p. Suppose firm j is employing the equilibrium strategy. Then, if firm i uses frame A and charges a price $p \in [p_0^A, p_1^A]$, its expected profit is

$$\pi(A, p) = p\{\lambda x_A(p) + (1-\lambda)[\alpha_1/2 + (1-\alpha_1) x_B(p)]\} . \tag{2}$$

With probability λ, the rival is also using A so that the firms compete *à la* Bertrand. With probability $1-\lambda$, the rival is using B, so that a fraction α_1 of the consumers are confused (by frame differentiation) and shop randomly, and the firms compete *à la* Bertrand for the remaining $1-\alpha_1$ fully aware consumers.

If instead firm i uses B and charges $p \in [p_0^B, p_1^B]$, its expected profit is

$$\pi(B, p) = p\{\lambda[\alpha_1/2 + (1-\alpha_1) x_A(p)] + (1-\lambda)[\alpha_2/2 + (1-\alpha_2) x_B(p)]\} . \tag{3}$$

With probability λ, the rival uses A so that a fraction α_1 of the consumers are confused (by frame differentiation) and shop randomly. With probability $1-\lambda$, the rival also uses B so that a fraction α_2 of the consumers are confused (by frame complexity) and shop randomly.[16]

The nature of the equilibrium depends on which confusion source dominates. Intuitively, when $\alpha_1 < \alpha_2$, if a firm shifts from frame A to B, more consumers get confused regardless of its rival's frame choice. Thus, each firm charges higher prices when it uses frame B than when it uses frame A. For $\alpha_1 > \alpha_2$, when a firm shifts from frame A to B, more consumers get confused if its rival uses A, while fewer consumers get confused if its rival uses B. Hence, there is no obvious monotonic relationship between the prices associated with A and B. Below we analyze these two cases separately.

- *Frame differentiation dominates frame complexity:* $0 \leq \alpha_2 < \alpha_1$

The unique symmetric equilibrium in this case dictates $F_A(p) = F_B(p)$ and $S_A = S_B = [p_0, 1]$ (see Appendix A for the proof). That is, a firm's price is independent of its frame. Let $F(p)$ be the common price cdf and $x(p) \equiv 1 - F(p)$. Then, using the profit functions (2) and (3) and the frame indifference condition $\pi(A,p) = \pi(B,p)$, we obtain

$$\lambda = 1 - \frac{\alpha_1}{2\alpha_1 - \alpha_2} \ . \tag{4}$$

When $\alpha_2 = 0$ (i.e., frame differentiation is the sole confusion source), firms are equally likely to adopt each frame (i.e., $\lambda = 1/2$). When $\alpha_2 > 0$ (i.e., frame complexity is also a confusion source), firms adopt it more often (i.e., $1-\lambda$ increases with α_2).

Note that (4) can be re-written as $(1-\lambda)\alpha_1 = \lambda\alpha_1 + (1-\lambda)\alpha_2$ and it actually requires the expected number of confused consumers to be the same when a firm uses frame A (the left-hand side) and when it uses frame B (the right-hand side). As in duopoly there are only two types of consumers (the confused and the fully aware), then the expected market composition along the equilibrium path does not depend on a firm's frame choice. Since the pricing balances the incentives to extract surplus from the confused and to compete for the fully aware, a frame-independent market composition implies frame-independent pricing. This explains why $F_A = F_B$. (This result may not hold if the confused are biased toward the simple frame as formally shown in the general oligopoly model.)

Let π be a firm's equilibrium profit. Since all prices on $[p_0, 1]$ should result in the same profit, we obtain (e.g. from $\pi(A,1)$ by using $x(1) = 0$)

$$\pi = \frac{\alpha_1^2}{2(2\alpha_1 - \alpha_2)} \ . \tag{5}$$

π increases with both α_1 and α_2. That is, confusion (regardless of its source) always boosts firms' payoffs and harms consumers.

Finally, the common price cdf $F(p)$ can be derived from the mixed-strategy equilibrium constant profit condition, $\pi(A,p) = \pi$. Explicitly, $x(p) = 1 - F(p)$ solves

$$\lambda x(p) + (1-\lambda)\left[\alpha_1/2 + (1-\alpha_1)x(p)\right] = \frac{\pi}{p} \ . \tag{6}$$

[16] Note that the profit functions apply for any price p as $F_z(p) = 0$ for $p < p_0^z$ and $F_z(p) = 1$ for $p > p_1^z$.

Then the boundary price p_0 is defined by $x(p_0) = 1$ and one can check that $p_0 \in (0, 1)$. The price cdf for a higher α_1 (α_2) first-order stochastically dominates that for a lower α_1 (α_2). This is consistent with the observation that confusion benefits firms and harms consumers. We summarize these findings below:

Proposition 1 *In the duopoly model with $0 \leq \alpha_2 < \alpha_1$, there is a unique symmetric mixed-strategy equilibrium where each firm adopts frame A with probability λ and frame B with probability $1 - \lambda$, and λ is given in (4). Regardless of its frame choice, each firm chooses its price randomly according to a cdf F which is defined by (6) on $[p_0, 1]$. Each firm's equilibrium profit is π given in (5).*

Notice that the equilibrium price dispersion is driven by firms' obfuscation effort through random framing but *not* necessarily by the coexistence of price aware and confused consumers. This is best seen in the polar case with $\alpha_1 = 1$ and $\alpha_2 = 0$, where consumers are always homogeneous both ex-ante and ex-post (i.e., once a pair of frames is realized, either all consumers are confused or all of them are fully aware), but price dispersion still persists.

- *Frame complexity dominates frame differentiation:* $0 < \alpha_1 < \alpha_2$

In this case, the unique symmetric equilibrium dictates adjacent supports $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$ (see Appendix A for the proof). In particular, if $\alpha_2 = 1$, then $S_A = [p_0^A, 1)$ and $S_B = \{1\}$. That is, frame B is always associated with higher prices than frame A. This happens because when a firm shifts from frame A to frame B, regardless of the rival's frame, more consumers get confused given that $\alpha_1 < \alpha_2$.

With adjacent price supports, in the profit function $\pi(A, p)$ (in expression (2)), $x_B(p) = 1$ for any $p \in S_A$ since frame B is always associated with higher prices. Similarly, in the profit function $\pi(B, p)$ (in expression (3)), $x_A(p) = 0$ for any $p \in S_B$. Then from the indifference condition $\pi(A, \hat{p}) = \pi(B, \hat{p})$, we can derive

$$\lambda = 1 - \frac{\alpha_1}{\alpha_2} \ . \tag{7}$$

Note that the probability of using the complex frame B $(1 - \lambda)$ *decreases* with the complexity index α_2, unlike the previous case (with $\alpha_1 > \alpha_2$). This happens because when confusion from frame complexity dominates, the prices associated with frame B are already high (so a rival using frame B is a softer competitor). This makes more attractive the use of frame A together with a relatively high price (but still lower than $\hat{p}$). Hence, for fixed α_1, the overall relationship between $1 - \lambda$ and α_2 is non-monotonic: when $\alpha_2 < \alpha_1$, the probability of using frame B rises with α_2 and when $\alpha_2 > \alpha_1$, it decreases with α_2.

Each firm's equilibrium profit π is given by $\pi(B, 1)$:

$$\pi = \alpha_1 (1 - \frac{\alpha_1}{2\alpha_2}) \ . \tag{8}$$

As before, it can be verified that this equilibrium profit increases (and so consumer surplus decreases) with both α_1 and α_2.

Finally, $F_z(p)$ is determined by $\pi(z,p) = \pi$. Explicitly, we have

$$\lambda x_A(p) + (1-\lambda)(1-\alpha_1/2) = \frac{\pi}{p} \tag{9}$$

and

$$\lambda\alpha_1/2 + (1-\lambda)[\alpha_2/2 + (1-\alpha_2)x_B(p)] = \frac{\pi}{p}\ . \tag{10}$$

The boundary prices p_0^A and $\hat{p}$ are defined by $x_A(p_0^A) = 1$ and $x_A(\hat{p}) = 0$, respectively. Both of them are well defined with $p_0^A < \hat{p}$. We summarize these results below:

Proposition 2 *In the duopoly model,*
(i) if $\alpha_1 < \alpha_2 < 1$, there is a unique symmetric mixed-strategy equilibrium where each firm adopts frame A with probability λ and frame B with probability $1-\lambda$, and λ is given in (7). When a firm uses frame A, it chooses its price randomly according to the cdf F_A defined on $[p_0^A, \hat{p}]$ which solves (9); when a firm uses frame B, the price cdf is F_B defined on $[\hat{p}, 1]$ which solves (10). Each firm's equilibrium profit π is given in (8).
(ii) if $\alpha_1 < \alpha_2 = 1$, the equilibrium has the same form except that F_B is a degenerate distribution on $\{1\}$ and F_A is defined on $[p_0^A, 1)$.

When $\alpha_2 \to \alpha_1$, Propositions 1 and 2 indicate that the firms use frame B almost surely ($\lambda \to 0$), and the price cdf's associated with B in the two cases tend to coincide. So, when $\alpha_1 = \alpha_2 > 0$, there is a unique symmetric equilibrium in which both firms use frame B.[17]

Our analysis focuses on cases where price framing is a short term decision. However, there are also cases where changes in price frames might take time or be costly (say, they require to redesign the contract form), whereas firms still can adjust prices frequently. In this scenario, it is more appropriate to consider a two-stage game where firms commit to frames before competing in prices. Below we discuss the equilibria in such a sequential-move variant of our duopoly model.

When $\alpha_1 > \alpha_2$, there are two pure-strategy equilibria where firms choose different frames (if they can coordinate successfully). In the second stage, firms mix on prices (as in part (c) in the proof of Lemma 1) and each firm earns $\alpha_1/2$. There is also an equilibrium where firms mix their frame choices in the first stage. This equilibrium is more likely when frame coordination is hard to achieve. More specifically, it can be shown that in the mixed-strategy equilibrium each firm adopts frame A with probability equal to (4) and makes profit equal to (5). That is, in the mixed-strategy equilibrium of the sequential-move game the frequency of using each frame and the welfare are the same as in the simultaneous-move game.

When $\alpha_1 < \alpha_2$, there is a unique equilibrium in which both firms adopt the complex frame B. This is qualitatively different from the simultaneous-move setting. The outcome in the

[17] Note that when $\alpha_1 = \alpha_2 > 0$, the frame profile (A, B) cannot form part of an equilibrium. Otherwise, the firm using frame B would have an incentive to switch to frame A and undercut its rival, as we argued in part (c) in the proof of Lemma 1.

pricing stage echoes part (b) in the proof of Lemma 1, and each firm makes $\alpha_2/2$ (which is greater than (8)). In sum, in a two-stage game, a pure-strategy equilibrium is more likely and firms tend to refrain from mixing on frames. But, there is still consumer confusion in the market either because firms adopt different frames or because they use complex frames.

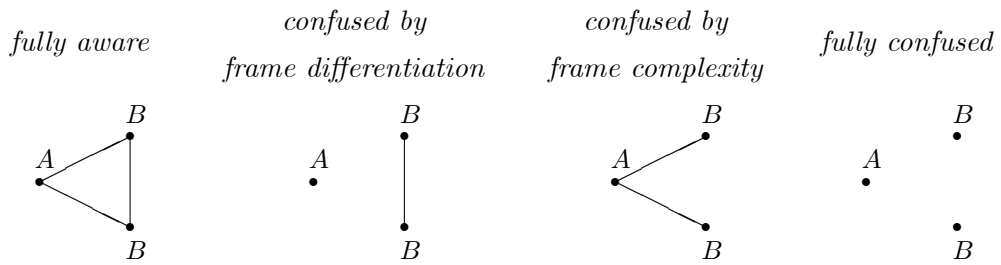
3 The Oligopoly Model

In this section, we develop a general oligopoly version of the model to analyze the impact of competition on market outcomes in the presence of price framing.

Consider a homogeneous product market with $n \geq 2$ identical sellers and, as before, two categories of frames, A and B. A is a simple frame so that all prices in this frame are comparable. B is potentially complex so that with probability $\alpha_2 \geq 0$ the consumers cannot compare prices in this frame. Consumers can also be confused by frame differentiation and so unable to compare prices in different frames with probability $\alpha_1 > 0$. In continuation, we focus on the case where confusion due to frame differentiation is independent of confusion due to frame complexity. However, depending on the microfoundations, the two types of confusion may be correlated. We argue in Section 4 that our analysis and its main insights carry over to the case where the two confusion sources are dependent.

In duopoly, for any realized frame profile, there is at most one confusion source, and so there are at most *two* types of consumers: fully aware (who buy the cheaper product) and totally confused (who shop randomly). With more than two firms, for a realized frame profile (e.g., (A, B, B)), both confusion sources might be present. So, there are up to *four* groups of consumers: $(1-\alpha_1)(1-\alpha_2)$ fully aware ones, $\alpha_1(1-\alpha_2)$ consumers confused only by frame differentiation, $(1-\alpha_1)\alpha_2$ consumers confused only by frame complexity, and $\alpha_1\alpha_2$ consumers confused by both confusion sources.[18] Below is an illustrative example.

Example 1 *Consider a case with* 3 *firms. Suppose firm 1 uses frame A, and firms 2 and 3 use frame B, respectively. The following graphs show the comparability among options for the four types of consumers. If two offers are comparable they are connected; if they are not comparable there is no link between them.*



Moreover, with more than two firms, even if there is only one confusion source, a consumer may be only partially confused as the following example shows.

[18] Note that in our model consumer confusion occurs at frame level. For example, across all pairs of one A and one B offer, a consumer is either able to compare all or none.

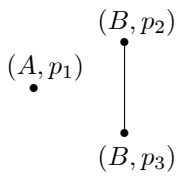
Example 2 *Consider a case with* 3 *firms. Firm 1 uses frame* A *and charges price* p_1*, and firms 2 and 3 use frame* B *and charge* p_2 *and* p_3*, respectively. If* $\alpha_1 = 1$ *and* $\alpha_2 = 0$ *(i.e., frame* B *is also simple), then only frame differentiation causes confusion. All consumers can accurately compare* p_2 *with* p_3 *since they are presented in the same frame, but cannot compare* p_1 *with either* p_2 *or* p_3*. So consumers are neither fully aware nor totally confused.*

So, a major question is how does a consumer choose from a "partially ordered" set in which some pairs of alternatives are comparable, but others are not. Note that this is not an issue in the duopoly model. To address this consumer choice issue, following the literature on incomplete preferences, we adopt a *dominance-based* consumer choice rule. The basic idea is that consumers only choose, according to some stochastic rule, from the "maximal" alternatives which are not dominated by any other comparable alternative. From now on, we use "dominated" in the following sense.

Definition 1 *For a consumer, firm* i*'s offer* $(z_i, p_i) \in \{A, B\} \times [0, 1]$ *is* ***dominated*** *if there exists firm* $j \neq i$ *which offers alternative* $(z_j, p_j < p_i)$ *and the two offers are comparable.*

Notice that for any consumer, the set of maximal or undominated alternatives is well-defined and non-empty (for example, the firm that charges the lowest price in the market is never dominated), and it can be constructed, for example, by conducting pairwise comparisons among all alternatives.[19] The example below illustrates our consumer choice rule.

Example 3 *Consider Example 2 and let* $p_1 < p_2 < p_3$*.*



Offers in frame B *are comparable since* B *is also a simple frame. Then firm 3's offer is dominated by firm 2's offer given that* $p_2 < p_3$*. Offers in different frames are not comparable since* $\alpha_1 = 1$*, so both firm 1's offer and firm 2's offer survive. Then, consumers buy from firm 1 with some probability and from firm 2 with the complementary probability.*

In this example, there is only one consumer group (all buyers are confused only by frame differentiation) so that all consumers face the same set of undominated offers. In general, when $\alpha_1, \alpha_2 \in (0, 1)$ the set of undominated alternatives varies across consumer groups.

Now we can formally state our *dominance-based* consumer choice rule as follows:

[19] In our model, the comparability of two offers is independent of their comparability with other available offers. This excludes transitivity of comparability. Consider a consumer who can compare offers in different frames, but cannot compare offers in frame B. Then the presence of an offer in frame A (which is comparable with any of the B offers) does not help the consumer compare offers in frame B directly. This might be the case when the consumers use different procedures to compare prices in different formats and to compare prices in a complex format.

1. Consumers first eliminate all dominated offers in the market.

2. They then buy from the undominated firms according to the following stochastic purchase rule (which is independent of prices): (i) if all these firms use the same frame, they share the market equally; (ii) if among them $n_A \geq 1$ firms use frame A and $n_B \geq 1$ firms use frame B, then each undominated A firm is chosen with probability $\phi(n_A, n_B)/n_A$ and each undominated B firm is chosen with probability $[1 - \phi(n_A, n_B)]/n_B$, where $\phi(\cdot) \in (0, 1)$ is non-decreasing in n_A and non-increasing in n_B and $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$.

Note that $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$ in 2(ii) allows the consumers to favor the simple frame A.[20] This generalizes the random purchase assumption in our duopoly example. $\phi(\cdot) < 1$ excludes the possibility that all consumers favor the simple frame.[21] The monotonicity assumption in 2(ii) means that the presence of more undominated firms with one frame increases the overall probability that consumers buy from them. Note that the uniformly random purchase rule $\phi(n_A, n_B) = n_A/(n_A + n_B)$ satisfies all the conditions.

For the rest of the paper, let

$$\phi_k \equiv \phi(1, k)$$

denote the probability that a consumer buys from the A firm when there are other k undominated B firms to choose from. Then, 2(ii) implies that $\{\phi_k\}_{k=1}^{n-1}$ is a non-increasing sequence: when more B firms survive, the undominated A firm has less demand, and $\phi_k \in [1/(1+k), 1)$. Note that $\phi_k = 1/(1+k)$ is the uniformly random purchase rule when consumers have no bias toward the simple frame.

Recall that in duopoly the type of market equilibrium depends on whether frame differentiation or frame complexity is more confusing. The same is true in the general case. Subsections 3.1 and 3.2 analyze the corresponding symmetric equilibrium and the impact of greater competition for $\alpha_1 < \alpha_2$ and $\alpha_1 > \alpha_2$, respectively.

Before we proceed, let us summarize two main findings. First, when $\alpha_2 > 0$ (i.e., when frame B is complex), greater competition tends to induce firms to use frame B more often. In particular, when there is a large number of firms, they use frame B almost surely. Intuitively, with more firms it becomes harder for them to frame differentiate, and so firms rely more on frame complexity to soften price competition. Second, when $\alpha_2 > 0$, industry profit is bounded away from zero even when there are an infinite number of firms, and greater competition can increase industry profit and harm consumers (i.e., consumers may actually pay more in a more competitive market).

[20] There is evidence that people have preferences for simpler options, especially when they face many alternatives. See, for instance, Iyengar and Kamenica (2010) and the references therein.

[21] For example, some consumers might be overconfident in their ability to compare offers, so they do not favor any particular frame, but may actually make mistakes.

3.1 Frame differentiation dominates frame complexity ($\alpha_1 > \alpha_2$)

We analyze now the case where consumers are more likely to be confused by frame differentiation than by the complexity of frame B (that is, $\alpha_1 > \alpha_2$). For simplicity, we first focus on the polar case in which prices in different frames are always incomparable (i.e., $\alpha_1 = 1$). We then discuss how the main results can be extended to the case with $\alpha_1 < 1$. All proofs missing from the text are relegated to Appendix B.1.

Lemma 4 in Appendix B.1 shows that there is no pure-strategy equilibrium when $\alpha_2 > 0$. If $\alpha_2 = 0$ (both frames are simple) and $n \geq 4$, there are always asymmetric pure-strategy equilibria in which each frame is used by more than one firm and all firms price at marginal cost. However, for any $n \geq 2$, there is a symmetric mixed-strategy equilibrium in which firms make positive profits.

A symmetric mixed-strategy equilibrium. Let (λ, F_A, F_B) be a symmetric mixed-strategy equilibrium, where λ is the probability of using frame A and F_z is a price cdf associated with frame $z \in \{A, B\}$. Let $[p_0^z, p_1^z]$ be the support of F_z. As in Lemma 3, it is clear that F_z is atomless everywhere (as now $\alpha_2 < 1$). For the rest of the paper,

$$P_{n-1}^k \equiv C_{n-1}^k \lambda^k (1-\lambda)^{n-k-1}$$

denotes the probability that k firms among $n-1$ ones adopt frame A at equilibrium, where C_{n-1}^k stands for combinations of $n-1$ taken k. Recall that $x_z(p) = 1 - F_z(p)$.

Along the equilibrium path, if firm i uses frame A and charges price p, its profit is:

$$\pi(A,p) = p\lambda^{n-1} x_A(p)^{n-1} + p \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2 \phi_{n-k-1} + (1-\alpha_2)\phi_1\right] . \tag{11}$$

If k other firms also use frame A, firm i has a positive demand only if all other A firms price higher than p. This happens with probability $x_A(p)^k$. Conditional on that, if there are no B firms in the market (if $k = n-1$), then firm i serves the whole market. The first term in $\pi(A,p)$ follows from this. Otherwise, firm i's demand depends on whether the consumer can compare the B firms' offers. If she is confused by frame complexity and unable to compare (which happens with probability α_2), all B firms are undominated (since no comparison between A and B is possible), and so firm i's demand is ϕ_{n-k-1}. If she is not confused by frame complexity and, therefore, can compare prices in frame B (this happens with probability $1-\alpha_2$), only one B firm is undominated and so firm i's demand is ϕ_1.

If instead, along the equilibrium path, firm i uses B and charges price p, its profit is:

$$\begin{aligned} \pi(B,p) &= p(1-\lambda)^{n-1}\left[\frac{\alpha_2}{n} + (1-\alpha_2) x_B(p)^{n-1}\right] \\ &\quad + p \sum_{k=1}^{n-1} P_{n-1}^k \left[\alpha_2 \frac{1-\phi_{n-k}}{n-k} + (1-\alpha_2)(1-\phi_1) x_B(p)^{n-k-1}\right] . \end{aligned} \tag{12}$$

The first term gives the expected profit when there are no A firms in the market: the consumers who are confused by frame complexity purchase randomly among all B firms, while those who

are not confused buy from firm i only if it offers the lowest price. When $k \geq 1$ firms use frame A (note that only one of them will be undominated), if the consumer is confused by frame complexity (i.e., unable to compare prices in frame B), all B firms are undominated and have demand $1 - \phi_{n-k}$ in total. Firm i shares equally this residual demand with the other B firms. If the consumer is not confused by frame complexity, to face a positive demand, firm i must charge the lowest price in group B (this happens with probability $x_B(p)^{n-k-1}$), in which case it gets the residual demand $1 - \phi_1$.

Note that for $\alpha_1 = 1$ price competition can only take place among firms that use the same frame, and so $x_A(p)$ does not appear in $\pi(B, p)$ and $x_B(p)$ does not appear in $\pi(A, p)$. This also implies that both profit functions are valid even if firm i charges an off-equilibrium price. Thus, the upper bounds of the price cdf's are frame-independent: $p_1^A = p_1^B = 1$. Otherwise any price greater than p_1^z would lead to a higher profit. Then the frame-indifference condition $\pi(A, 1) = \pi(B, 1)$, pins down a unique well-defined $\lambda \in (0, 1)$. (See equation (17) in Appendix B.1). Each firm's equilibrium profit is

$$\pi = \pi(A, 1) = (1 - \lambda)^{n-1}[\alpha_2 \phi_{n-1} + (1 - \alpha_2)\phi_1] \ . \tag{13}$$

The price distributions F_A and F_B are implicitly determined by $\pi(z, p) = \pi$ since any price in the support of F_z should lead to the same profit in a mixed-strategy equilibrium. Both F_z are uniquely defined. The boundary prices $p_0^z < 1$ are determined by $\pi(z, p_0^z) = \pi$. Deviations to prices lower than p_0^z are not profitable as they only result in a price loss and no demand increase. We characterize the symmetric equilibrium below.

Proposition 3 *For $n \geq 2$ and $\alpha_2 < \alpha_1 = 1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability λ and frame B with probability $1 - \lambda$. When a firm uses frame $z \in \{A, B\}$, it chooses its price randomly according to a cdf F_z defined on $[p_0^z, 1]$, and implicitly determined by $\pi(z, p) = \pi$ with $\pi(z, p)$ given in (11) and (12) and π given in (13).*

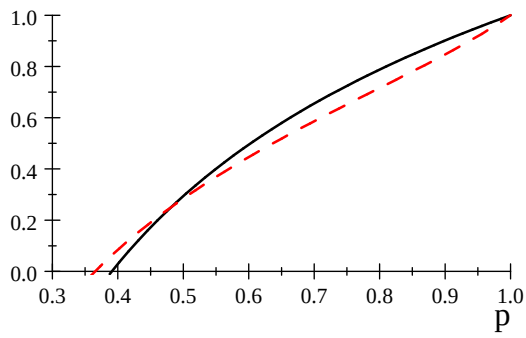


Figure 1: Price distributions with $n = 3$, $\alpha_1 = 1$ and $\alpha_2 = 0.5$

Figure 1 shows the equilibrium price distributions $F_A(p)$ (the solid line) and $F_B(p)$ (the dashed line) in the case with $n = 3$, $\alpha_1 = 1$, $\alpha_2 = 0.5$, and $\phi_k = 1/(1 + k)$.

Recall that in the duopoly equilibrium in Proposition 1 pricing is frame independent (i.e., $F_A(p) = F_B(p)$) if confused consumers have no exogenous bias toward a specific frame. However, this is no longer true in the case with more than two firms, as the above example indicates. (See Appendix B.1 for a rigorous treatment of this issue.) When $F_A \neq F_B$, there is no straightforward way to analytically rank the prices associated with the two frames.

The impact of greater competition. We now study the impact of an increase in the number of firms on the equilibrium framing strategies, and on profits and consumer surplus. Our analysis is based on the equilibrium characterized in Proposition 3. We first consider a market with many sellers, which provides the key insight for our main result.

Proposition 4 *When there are a large number of firms in the market,*

$$\lim_{n\to\infty} \lambda = \begin{cases} 1/2, & \text{if } \alpha_2 = 0 \\ 0, & \text{if } \alpha_2 > 0 \end{cases} \quad ; \quad \lim_{n\to\infty} n\pi = \begin{cases} 0, & \text{if } \alpha_2 = 0 \\ > 0, & \text{if } \alpha_2 > 0 \end{cases} .$$

When frame B is also a simple frame, the only way to reduce price competition is by frame differentiation. This is why in a sufficiently competitive market λ tends to $1/2$, which maximizes frame differentiation. However, the ability of frame differentiation alone to weaken price competition is limited. In fragmented markets, each frame is adopted by more than one firm almost surely (as long as λ is bounded away from zero and one), so price competition becomes extremely intense and the market price tends to marginal cost.

When frame B is complex, the impact of greater competition on firms' framing strategies changes completely. In a sufficiently competitive market, firms use frame B almost surely: they rely on frame complexity to soften price competition. (This is true even if frame B is only slightly more complex than frame A.) The reason is that, in a large market, the role of frame differentiation in reducing price competition becomes negligible, but the effect of frame complexity is still significant. For example, if all firms employ frame B for sure, industry profit is always α_2, regardless of the number of firms in the market. Hence, when frame B is complex, competition does not drive the market price to marginal cost. Note that these results hold even if some (but not all) confused consumers favor the simple frame.

The analysis for large n suggests that, when the number of firms increases, frame B's complexity becomes a more important anti-competitive device. In effect, as we show below, λ tends to decrease in the number of firms. That is, greater competition tends to induce firms to use the complex frame more frequently. Is it then possible that, in the presence of a complex frame B, greater competition raises market prices by increasing market complexity? The answer, in general, depends on the parameter values. But, at least for sufficiently large α_2, greater competition can actually increase industry profit and harm consumers. Therefore, in the market with price framing, competition policy which focuses exclusively on an increase in the number of competitors, might have undesired effects. For tractability, we focus on the uniformly random purchase rule $\phi_k = 1/(1+k)$.

Proposition 5 *With $0 < \alpha_2 < \alpha_1 = 1$ and the random purchase rule $\phi_k = 1/(1+k)$,*

(i) when n increases from 2 to 3, both λ and industry profit $n\pi$ decrease;
(ii) for any $n \geq 3$, there exists $\hat{\alpha} \in (0,1)$ such that for $\alpha_2 > \hat{\alpha}$, λ decreases but industry profit $n\pi$ increases from n to $n+1$.

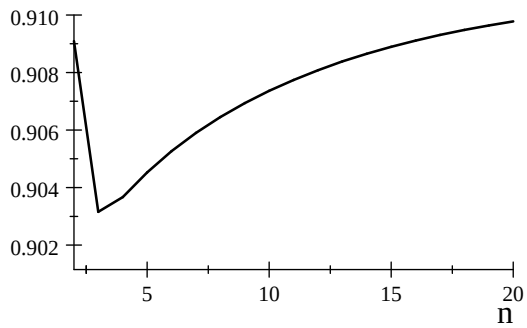


Figure 2: Industry profit and n when
$\alpha_1 = 1$ and $\alpha_2 = 0.9$

Beyond the limit results, numerical simulations suggest that λ tends to decrease in n, and industry profit can increase in n for a relatively large α_2.[22] Figure 2 shows how industry profit varies with n when $\alpha_2 = 0.9$.

The case with $\alpha_2 < \alpha_1 < 1$. Price competition can also take place between firms using different frames. Then both $x_A(p)$ and $x_B(p)$ appear in the profit functions $\pi(z,p)$. The more involved related analysis is presented in the supplementary document. There we show that if a symmetric mixed-strategy equilibrium exists, then it still satisfies $p_1^A = p_1^B = 1$. Numerical simulations suggest that greater competition can still have undesired effects (for example, when α_1 is large and α_2 is close to α_1). For example, when $\alpha_1 = 0.98$ and $\alpha_2 = 0.9$, industry profit varies with n in a way similar to Figure 2.

3.2 Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$)

Consider the case where consumers are more likely to be confused by the complexity of frame B than by frame differentiation (i.e., $\alpha_2 > \alpha_1$). Again, we first analyze the polar case in which prices in frame B are always incomparable (i.e., $\alpha_2 = 1$). We then discuss the robustness of our main results to the case with $\alpha_2 < 1$. The analysis resembles the previous one, so we only report the main results here and relegate the details to Appendix B.2.

Proposition 6 *For $n \geq 2$ and $0 < \alpha_1 < \alpha_2 = 1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability λ and frame B with probability $1-\lambda$. When a firm uses frame A, it chooses its price randomly according to a cdf F_A defined on $[p_0^A, 1)$; when it uses frame B, it charges a deterministic price $p = 1$.*

[22] For a sufficiently small α_2, increasing the number of firms will lower industry profit. This can be seen when $\alpha_2 = 0$, as $\lambda = 1/2$ (for any n) and industry profit is $n/2^n$, which decreases in n.

Using the equilibrium in proposition 6, we analyze the impact of greater competition on the market outcome. When there are many sellers in the market, the same results as in Proposition 4 for $\alpha_2 > 0$ hold. That is, $\lim_{n\to\infty} \lambda = 0$ and $\lim_{n\to\infty} n\pi > 0$. The same intuition applies: in a sufficiently competitive market, the ability of frame differentiation to soften price competition is negligible, and so firms resort to the complexity of frame B.

The following result shows that in the current case greater competition can also improve industry profit and decrease consumer surplus. In particular, this must happen when α_1 is small. The reason is that, for a small α_1, the complexity of frame B is more effective in reducing price competition, which makes the frequency of using frame B increase fast enough with the number of firms. The resulting market complexity could then dominate the usual competitive effect of larger n. Figure 3 below illustrates how industry profit varies with n when $\alpha_1 = 0.05$.[23]

Proposition 7 *In the case with $0 < \alpha_1 < \alpha_2 = 1$, for any $n \geq 2$, there exists $\hat{\alpha} \in (0, 1)$ such that for $\alpha_1 < \hat{\alpha}$, λ decreases while industry profit $n\pi$ increases from n to $n + 1$.*

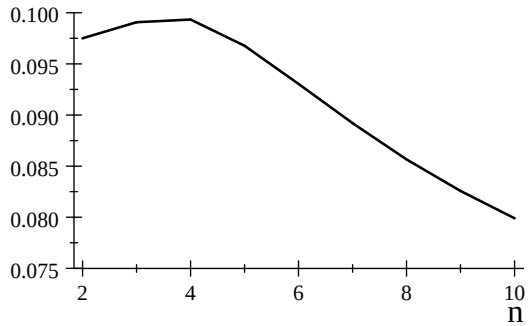


Figure 3: Industry profit and n when
$\alpha_1 = 0.05$ and $\alpha_2 = 1$

The case with $\alpha_1 < \alpha_2 < 1$. This analysis is more involved, and we relegate it to the supplementary document. Note that a symmetric separating equilibrium with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, p_1^B]$, resembling the one in Proposition 6, still exists under some parameter restrictions (when α_1 is not too close to $\alpha_2 < 1$). Also, for fixed $\alpha_2 < 1$, if α_1 is sufficiently small, greater competition can still increase industry profit and harm consumers.

4 Discussion

Comparison with the default-bias choice rule in Piccione and Spiegler (2012). The dominance-based choice rule embeds a *simultaneous* assessment of competing offers, and a consumer's final choice is not affected by the sequence of pairwise comparisons. This "simultaneous search" feature is suitable in markets where the consumers are not influenced by past experiences (or are

[23] For industry profit to increase at a larger n, α_1 needs to be smaller. But this is always feasible according to Proposition 7.

newcomers). Piccione and Spiegler (2012) consider a default-bias model where consumers are initially randomly attached to one brand (the default option), and they shift to another brand only if it is comparable to and better than their default. There, with sequential comparisons, a consumer's final choice depends on her default option.

In duopoly, the default-bias model is equivalent to the simultaneous assessment one (with the random purchase rule for confused consumers).[24] This is because, if the two firms' offers are comparable, in both models the better one attracts all consumers, whereas if they are incomparable, in both models the firms share the market equally. But, with more than two firms, the two models diverge. In this case, the default-bias model calls for further structure on the choice rule. To see why, consider the following example.

Example 4 *There are three firms in the market. Let $\alpha_2 = 1$ and $\alpha_1 = 0$ (the only confusion source is frame complexity). Firm 1 adopts frame A and prices at p_1, while firms 2 and 3 adopt frame B and price at p_2 and p_3, respectively, with $p_2 < p_1 < p_3$.*

The dominance-based rule implies that consumers purchase only from firm 2 since firm 3 is dominated by firm 1 and firm 1 is dominated by firm 2. Now consider the default-bias model. A consumer initially attached to firm 2 does not switch. If she is initially attached to firm 1, she switches to firm 2. However, if she is initially attached to firm 3, she switches to firm 1, but whether she further switches to firm 2 depends on what the choice rule of the default-biased consumer dictates. The rule should specify if the consumer assesses firm 2's offer using her default option (i.e., firm 3) or using her new choice (i.e., firm 2). By contrast, the dominance-based rule applies regardless of the number of firms in the market.[25]

A default bias adds another type of bounded rationality to confusion caused by framing. In this sense, our framework is a minimal deviation from the rational benchmark.

Noisy price comparisons. Our framework is a tractable way to capture the idea that confusion in price comparisons reduces consumers' price sensitivity. In particular, α_1 can be regarded as a measure of price elasticity reduction when consumers face prices in two different frames, and α_2 can be regarded as a measure of price elasticity reduction when consumers face two offers in frame B. An alternative (but less tractable) way to model the framing effect is by introducing noisy price comparisons. If firm i adopts the simple frame A, consumers understand its price p_i perfectly. Instead, if it adopts a complex frame B, consumers perceive its price as $p_i + \varepsilon_i$. This allows consumer choices to depend on the price difference between two products even when consumers get confused. In this alternative model, if both ε_i and ε_j have a symmetric distribution around zero, it can be shown that in the symmetric equilibrium the

[24] More precisely, the equivalence requires the probability of being confused by two frames to be independent of which one is the default option.

[25] The fact that these two choice rules may lead to different outcomes can also be seen from the following example: consider the frame choices in example 4, but let $\alpha_2 = 0$ and $\alpha_1 = 1$, and $p_1 < p_2 < p_3$. Our approach (with the uniform purchase rule) predicts that firms 1 and 2 will share the market equally; while the default-bias rule predicts that firm 1 has demand $\frac{1}{3}$ and firm 2 has demand $\frac{2}{3}$.

firms still randomize on both frames and prices (see the supplementary document for details). However, it is not possible to fully characterize the equilibrium.

Both cases $\alpha_1 > \alpha_2$ and $\alpha_1 < \alpha_2$ can be justified in this setting with noisy price comparisons. To illustrate, suppose ε_i is a random variable with the standard normal distribution Φ. When both firm i and firm j use frame B, suppose $(\varepsilon_i, \varepsilon_j)$ follow a joint normal distribution with correlation coefficient $\rho \in [0, 1]$. Then $\varepsilon_i - \varepsilon_j$ follows a normal distribution with zero mean and variance $2(1 - \rho)$.

When both firms adopt frame A, demand is perfectly elastic at $p_i = p_j$. When only one firm, say, firm i, adopts frame B, its demand function is

$$Q_i = \Pr(p_i + \varepsilon_i < p_j) = \Phi(p_j - p_i) \ .$$

So demand elasticity at $p_i = p_j$ is

$$2p_i\phi(0) \ , \tag{14}$$

where $\phi(\cdot)$ is the standard normal density. When both firms adopt frame B, firm i's demand function is

$$Q_i = \Pr(p_i + \varepsilon_i < p_j + \varepsilon_j) = \Phi(\frac{p_j - p_i}{\sqrt{2(1-\rho)}}) \ ,$$

and so demand elasticity at $p_i = p_j$ is

$$2p_i \frac{\phi(0)}{\sqrt{2(1-\rho)}} \ . \tag{15}$$

If $\rho < \frac{1}{2}$ (e.g., if ε_i is independent of ε_j), (15) is less than (14), so the demand is less elastic when both firms adopt frame B than when only one firm does so. (This corresponds to the case of $\alpha_1 < \alpha_2$.) In contrast, if $\rho > \frac{1}{2}$, the opposite is true. (This corresponds to the case of $\alpha_1 > \alpha_2$.) In particular, when the two error terms ε_i and ε_j are perfectly correlated, frame B can be regarded as a simple frame and we return to the case with perfectly elastic demand. The correlation between ε_i and ε_j might be affected by how frame similarity influences consumer misperception. If a consumer misperceives two prices in frame B in a similar way (e.g., underestimates them to a similar extent), then the correlation between ε_i and ε_j should be high.

Costly information processing as an alternative interpretation. Our model assumes that there are boundedly rational consumers who are unable to compare framed prices or understand the market equilibrium. But it can also be interpreted as a model with rational consumers and costly information processing. Price comparisons in the presence of framing might require costly information processing, and consumers may differ in their costs. Then, some consumers who have high information processing costs will opt out of doing so, and just behave as the confused consumers in our model.[26]

[26] Price framing reduces the comparability of competing offers and increases consumers' search/evaluation costs. In a related vein, Kuksov (2004) presents a consumer search model where firms produce more differentiated products in response to lower search costs. As a result, lower search costs may lead to higher prices in the market. In Kuksov and Villas-Boas (2010), firms' range of alternatives affects consumers' search/evaluation costs and too many alternatives may induce consumers to leave the market without making a choice.

However, an interpretation with rational consumers might be inconsistent with the separating equilibrium in Proposition 2 (where the complex frame is always associated with higher prices than the simple one). Rational consumers should be able to infer prices from frames and always choose the simple-frame product.[27] Then, the separating equilibrium would not be valid. (This is not an issue in our model with boundedly rational consumers.) Nevertheless, notice that the separating equilibrium could still make sense if there is always a non-trivial mass of naive consumers who do not try to understand market equilibrium.

Carlin (2009) considers a setting related to our case with $\alpha_2 > \alpha_1$. In his model, if a consumer incurs a cost, she can learn all prices in the market, thereby purchasing the cheapest product; otherwise, she remains uninformed and shops randomly. In equilibrium, higher complexity is associated with higher prices. Consumers in Carlin's model cannot infer prices from a firm's price complexity level because they cannot observe individual firms' complexities but only observe the aggregate market complexity.

Dependence between the two sources of confusion. In our oligopoly model in Section 3, we assumed that confusion due to frame differentiation and confusion due to frame complexity are independent and considered up to four types of consumer groups whose sizes are determined by the parameters α_1 and α_2. However, the two sources of confusion may be dependent. Take, for instance, our numeracy-skill example for $\alpha_1 > \alpha_2$ in subsection 2.1. There, confusion stems from poor numeracy skills and it is mitigated by similarity. So if a consumer is confused by two complex frames, she must also be confused by two different frames.

To allow for dependence between the two sources of confusion, we can regard the four consumers groups as the primitives of the model. A fraction α_{FD} of consumers are confused only by frame differentiation, a fraction α_{FC} of consumers are confused only by frame complexity, a fraction α_B are confused by either source, and the remaining fraction $1 - \alpha_{FD} - \alpha_{FC} - \alpha_B$ of consumers are fully aware. (Note that the two confusion sources are independent if and only if $\alpha_{FD} = \alpha_1(1 - \alpha_2)$, $\alpha_{FC} = \alpha_2(1 - \alpha_1)$ and $\alpha_B = \alpha_1\alpha_2$.) Then, our analysis carries over with some change of notation.[28] In particular, the case with $\alpha_1 = 1 > \alpha_2$ analyzed in Section 3.1 corresponds to $\alpha_{FD} = 1 - \alpha_2$ and $\alpha_B = \alpha_2$, and the case with $\alpha_2 = 1 > \alpha_1$ analyzed in Section 3.2 corresponds to $\alpha_{FC} = 1 - \alpha_1$ and $\alpha_B = \alpha_1$. In these two polar cases, (in)dependence of the two confusion sources actually does not play a role.

5 Conclusion

This paper presents a model of competition in both prices and price frames. In a homogeneous product market, price framing can obstruct consumers' ability to compare prices and result in confusion. Our study shows that in the symmetric equilibrium firms randomize on both price

[27] In this sense, our assumption that consumers weakly favor the simple frame (i.e., $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$) partially reflects such sophistication.

[28] The analysis in the duopoly example also remains unchanged as long as we replace α_1 by $\alpha_{FD} + \alpha_B$ and α_2 by $\alpha_{FC} + \alpha_B$.

frames and prices, and make positive profits. An increase in the number of firms reduces firms' ability to frame differentiate and makes them use complex frames more often. As a result, greater competition might increase profits and harm consumers. In our setting, consumer confusion may stem from price format incompatibility or price complexity. The nature of the equilibrium depends on which source of consumer confusion leads to more confused consumers.

This study is motivated by price framing, but it also applies to situations where *product framing* reduces the comparability of offers. For instance, the way of presenting nutritional information might frame identical food products differently. An "improved recipe" or a "British meal" label might spuriously differentiate a ready meal from its close substitutes.[29] Package size differences or quantity premia could also make it harder to compare products. On the same supermarket shelf toothpastes come in tubes of 50, 75 or 100 ml, and refreshments, cleaning products, tea boxes occasionally come in larger -"extra 25% free"- containers. This interpretation also relates our paper to the literature on endogenous product differentiation (see, for instance, Chapter 7 in Tirole, 1988). A main difference is that in our model firms make product differentiation and price choices simultaneously.

Our model predicts that firms randomize their frame choices in order to obstruct consumer price comparisons. But in some markets, changing frames frequently could be costly and, as a result, firms may refrain from mixing on price frames. In addition, ex post consumers might be able to find out the actual prices they paid, even if they were confused at the time of purchase. In that case, if they discover that they were misled by a firm's price framing strategy, they may avoid this firm in the future. This reputational concern would also reduce firms' incentive to frame prices.

Finally, in financial, energy, or mortgage markets amongst others, information intermediaries could help consumers compare offers and identify the best deals. Their presence should mitigate confusion caused by price framing and thus reduce firms' incentives to frame their prices. However, in spite of this, consumer confusion seems to persist in these environments. There might be several reasons why information intermediaries do not completely solve the issue of price framing and consumer confusion. On the demand side, take-up of such services is not universal. Consumers differ in their opportunity costs of time or search methods and they may rationally opt out of using such services. They may also overestimate their ability to make accurate comparisons without specialized help. In some markets where the best tariff choice depends on individual characteristics, consumers might be reluctant to share the relevant information with intermediaries due to privacy concerns. On the supply side, information intermediaries have incentives to strategically limit sellers' participation to such platforms (e.g., by charging high participation fees) in order to protect their informational value and ability to extract rents.

[29] The reportage "What's really in our food?" broadcast on BBC One in July 2009 stressed this point. Interviewed customers admitted to being misled by a ready food made with imported meat and labeled as "British meal". Also, buyers seem to have a poor understanding of what labels such as "free range" really mean.

A Appendix: Proofs in the Duopoly Case

Proof of Lemma 3: Suppose that F_A has a mass point at some price $p \in S_A$ in the symmetric equilibrium. Equilibrium condition requires that (A, p) generates the same profit as other frame and price combinations. Given the mass point, in the symmetric equilibrium there is a positive probability that both firms use frame A and tie at p. In that event, since all consumers are price aware, reducing price slightly can increase demand discontinuously. This implies that a unilateral deviation to $(A, p - \varepsilon)$ (for a sufficiently small $\varepsilon > 0$) generates a higher profit than (A, p), which leads to a contradiction.

A similar argument works for frame B. Suppose that F_B has a mass point at some price $p \in S_B$ in the symmetric equilibrium. Equilibrium condition requires that (B, p) generates the same profit as other frame and price combinations. Given the mass point, in the symmetric equilibrium there is a positive probability that both firms use frame B and tie at p. In that event, since some consumers are price aware when $\alpha_2 < 1$, reducing price slightly can increase demand discontinuously. This implies that a unilateral deviation to $(B, p - \varepsilon)$ (for a sufficiently small $\varepsilon > 0$) generates a higher profit than (B, p), which also leads to a contradiction.

Proof of Proposition 1: The proposed configuration is indeed an equilibrium since no deviation to $p < p_0$ is profitable. We show now that it is the unique symmetric mixed-strategy equilibrium with F_z strictly increasing on its support. Recall that, by Lemma 3, when $\alpha_2 < 1$, in any symmetric mixed-strategy equilibrium F_z is continuous on S_z. The proof entails several steps.

Step 1: $S_A \cap S_B \neq \emptyset$. Suppose $p_1^A < p_0^B$. Then if a firm uses frame A and charges p_1^A, its profit is

$$\pi(A, p_1^A) = p_1^A (1 - \lambda) [(1 - \alpha_1) + \alpha_1/2] \ .$$

The firm has positive demand only if the rival uses frame B, in which case it sells to all price aware consumers and to half of the confused ones. Clearly, this firm can do better by charging a price slightly higher than p_1^A. A contradiction. Similarly, we can rule out the possibility of $p_1^B < p_0^A$.

Step 2: $\max\{p_1^A, p_1^B\} = 1$. Suppose $p_1^z = \max\{p_1^A, p_1^B\} < 1$. Then, p_1^z is dominated by $p_1^z + \varepsilon$ (for some small $\varepsilon > 0$).

Step 3: $S_A = S_B = [p_0, 1]$. Suppose $p_1^A < p_1^B = 1$. Then, along the equilibrium path, if firm i uses frame A and charges $p \in [p_1^A, 1]$, its profit is

$$\pi(A, p) = p (1 - \lambda) [(1 - \alpha_1) x_B(p) + \alpha_1/2] \ ,$$

since it faces a positive demand only if firm j uses frame B. If firm i uses frame B and charges the same price p, its profit is

$$\pi(B, p) = p \{\lambda \alpha_1/2 + (1 - \lambda) [(1 - \alpha_2) x_B(p) + \alpha_2/2]\} \ ,$$

that should be equal to the candidate equilibrium profit. As the supposition $p_1^A < p_1^B = 1$ and Step 1 imply that $p_1^A \in S_B$, the indifference condition requires $\pi(A, p_1^A) = \pi(B, p_1^A)$ or

$$(1-\lambda)(\alpha_1 - \alpha_2) - \lambda\alpha_1 = 2(1-\lambda)(\alpha_1 - \alpha_2)\, x_B(p_1^A) \ .$$

But, if this equation holds, $\pi(A,p) > \pi(B,p)$ for $p \in (p_1^A, 1]$ as $\alpha_1 > \alpha_2$ and x_B is strictly decreasing on S_B. A contradiction. Similarly, we can exclude the possibility of $p_1^B < p_1^A = 1$. Hence, it must be that $p_1^A = p_1^B = 1$.

Then, from $\pi(A,1) = \pi(B,1)$, it follows that

$$\lambda\alpha_1 = (1-\lambda)(\alpha_1 - \alpha_2) \ . \tag{16}$$

Now suppose $p_0^A < p_0^B$. Then

$$\pi(A, p_0^B) = p_0^B[\lambda x_A(p_0^B) + (1-\lambda)(1-\alpha_1/2)] \text{ and}$$
$$\pi(B, p_0^B) = p_0^B\{\lambda[(1-\alpha_1)\, x_A(p_0^B) + \alpha_1/2] + (1-\lambda)(1-\alpha_2/2)\} \ .$$

Since the supposition $p_0^A < p_0^B$ and Step 1 imply that $p_0^B \in S_A$, we need $\pi(A, p_0^B) = \pi(B, p_0^B)$, or

$$2x_A(p_0^B) = 1 + \frac{1-\lambda}{\lambda}\frac{\alpha_1 - \alpha_2}{\alpha_1} \ .$$

The left-hand side is strictly lower than 2 given that x_A is strictly decreasing on S_A and $p_0^A < p_0^B$. But (16) implies that the right-hand side is equal to 2. A contradiction. Similarly, we can exclude the possibility of $p_0^A < p_0^B$. Hence, it must be that $p_0^A = p_0^B$.

Step 4: $F_A = F_B$. For any $p \in [p_0, 1]$, the indifference condition requires $\pi(A,p) = \pi(B,p)$. Using (2) and (3), we get

$$\lambda\alpha_1[x_A(p) - 1/2] = (1-\lambda)(\alpha_1 - \alpha_2)[x_B(p) - 1/2]$$

for all $p \in [p_0, 1]$. Then (16) implies $x_A = x_B$ (or $F_A = F_B$).

Proof of Proposition 2: (1) Let us first prove the result for $\alpha_2 < 1$.
(1-1) A deviation to $(A, p < p_0^A)$ is obviously not profitable. A deviation to $(A, p > \hat{p})$ generates a profit equal to

$$p(1-\lambda)[(1-\alpha_1)\, x_B(p) + \alpha_1/2] \ .$$

Using (7), one can check that the deviation profit is lower than $\pi(B,p)$ in (3) with $x_A(p) = 0$. The last possible deviation is $(B, p < \hat{p})$ that results in a profit equal to

$$p\{\lambda[(1-\alpha_1)\, x_A(p) + \alpha_1/2] + (1-\lambda)(1-\alpha_2/2)\} \ .$$

Again, using (7), one can check that the deviation profit is lower than $\pi(A,p)$ in (2) with $x_B(p) = 1$.

(1-2) We now prove uniqueness. As in the proof of Proposition 1, we can show that $S_A \cap S_B \neq \emptyset$ and $\max\{p_1^A, p_1^B\} = 1$. Then the two steps below complete the proof.

Step 1: $S_A \cap S_B = \{\hat{p}\}$ for some $\hat{p}$. Suppose to the contrary that $S_A \cap S_B = [p', p'']$ with $p' < p''$. Then for any $p \in [p', p'']$, it must be that $\pi(A, p) = \pi(B, p)$, where the profit functions are given by (2) and (3). This indifference condition requires that

$$\lambda\alpha_1 [x_A(p) - 1/2] = (1 - \lambda)(\alpha_1 - \alpha_2)[x_B(p) - 1/2]$$

for all $p \in [p', p'']$. Since $\alpha_1 < \alpha_2$ and F_z is strictly increasing on S_z, the left-hand side is a decreasing function of p, while the right-hand side is an increasing function of p. So the condition cannot hold for all $p \in [p', p'']$. A contradiction.

Step 2: $p_1^B = 1$. Suppose $p_1^B < 1$. Then Step 1 and $\max\{p_1^A, p_1^B\} = 1$ imply that $p_1^A = 1$ and $p_1^B = p_0^A = \hat{p} < 1$. Then each firm's equilibrium profit should be equal to $\pi(A, 1) = (1 - \lambda)\alpha_1/2$ since the prices associated with B are lower than one. But, if a firm chooses frame B and $p = 1$, its profit is $[\lambda\alpha_1 + (1 - \lambda)\alpha_2]/2$ since it sells to half of the confused consumers. This deviation profit is greater than $\pi(A, 1)$ given that $\alpha_2 > \alpha_1$. A contradiction.

Therefore, in equilibrium, it must be that $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$.

(2) The equilibrium when $\alpha_2 = 1$ is just the limit of the equilibrium in (1) as $\alpha_2 \to 1$. But, now $S_A = [p_0^A, 1)$ and $S_B = \{1\}$.

B Appendix: Proofs and Omitted Details in the Oligopoly Model

B.1 The case with $0 < \alpha_2 < \alpha_1 = 1$

Lemma 4 *In the oligopoly model with $0 < \alpha_2 < \alpha_1 = 1$, there is no equilibrium in which all firms adopt deterministic frames.*

Proof. We prove this lemma in three steps:

(a) In any possible equilibrium where firms use deterministic frames, at most one firm uses frame A. Suppose to the contrary that at least two firms use A. Then they must all earn zero profit at any putative equilibrium. But then any of them has unilateral incentives to deviate to frame B and a positive price, to make a positive profit as $\alpha_2 > 0$. A contradiction.

(b) In any possible equilibrium where firms use deterministic frames, at least one firm uses frame A. Suppose to the contrary that all firms use B. Then with probability α_2 consumers shop randomly, and with probability $1 - \alpha_2$ they buy from the cheapest firm. This is a version of Varian (1980), and each firm earns α_2/n.[30] But then any firm can earn more by deviating to frame A and price $p = 1$, which yields a profit of at least $\phi_{n-1} \geq 1/n$. This is because at most $n - 1$ B firms can survive and the deviator is never dominated as $\alpha_1 = 1$.

(c) Consider a candidate equilibrium where one firm uses A and all other firms use B.[31] First, the A firm must charge price $p = 1$ given that $\alpha_1 = 1$ and make a profit at least equal to ϕ_{n-1}.

[30] For $n \geq 3$, there are both symmetric and asymmetric mixed-strategy equilibria in the Varian model, but all of them are outcome equivalent (Baye et al., 1992).

[31] This part of the proof is different from that in the duopoly case since it is hard to directly characterize the pricing equilibrium when one firm uses frame A and other $n - 1 \geq 2$ firms use frame B.

Second, each B firm must also earn at least ϕ_{n-1}. Otherwise, any B firm that earns $\pi_B < \phi_{n-1}$ can improve its profit by deviating to frame A and a price $1-\varepsilon$ for small ε. (The deviator would make a profit at least equal to $(1-\varepsilon)\,\phi_{n-2}$ which is greater than π_B for a sufficiently small ε given that $\phi_{n-2} \geq \phi_{n-1}$.) Then, if $\phi_{n-1} > 1/n$, the sum of all firms' profits exceeds one, and we reached a contradiction since industry profit is bounded by one. The only remaining possibility is that $\phi_{n-1} = 1/n$ and each firm earns exactly $1/n$. But, then all firms charge the monopoly price $p = 1$[32] and any B firm has incentives to deviate to a price slightly below one given that $\alpha_2 < 1$. A contradiction. ■

Equilibrium condition for λ when $0 < \alpha_2 < \alpha_1 = 1$:

Since the price distributions for frames A and B share the same upper bound $p = 1$, letting $p = 1$ in (11)–(12) yields a frame indifference condition $\pi(A,1) = \pi(B,1)$. By dividing each side by $(1-\lambda)^{n-1}$ and rearranging the equation, we obtain

$$\alpha_2\left(\phi_{n-1} - \frac{1}{n}\right) + (1-\alpha_2)\,\phi_1 = \alpha_2 \sum_{k=1}^{n-2} \frac{C_{n-1}^{k}\left(1-\phi_{n-k}\right)}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^{k} + (1-\phi_1)\left(\frac{\lambda}{1-\lambda}\right)^{n-1}. \tag{17}$$

The right-hand side of (17) increases in $\lambda \in [0,1]$ from zero to infinity, and the left-hand side is positive for any $\alpha_2 \in [0,1)$ as $\phi_{n-1} \geq 1/n$. Hence, (17) has a unique solution λ in $(0,1)$ as we claimed in the main text.

The (im)possibility of price-frame independence. Recall that in the duopoly equilibrium in Proposition 1 pricing is frame independent (i.e., $F_A(p) = F_B(p)$). This feature of the duopoly equilibrium does not carry over to the oligopoly case. With duopoly, there are only two types of consumers. When $\alpha_2 < \alpha_1$, the equilibrium λ ensures that regardless of its frame choice a firm faces the same market composition if consumers have no exogenous bias toward a particular frame. This underlies price-frame independence. With more than two firms, there are in general more than two types of consumers. Although equilibrium λ ensures that the expected number of consumers who are totally insensitive to a firm's price is the same regardless of this firm's frame choice (i.e., $\pi(A,1) = \pi(B,1)$), this no longer guarantees that this firm also faces the same expected number of other types of consumers. In general, it is impossible for a firm to face the same market composition when it shifts from one frame to the other, and so its pricing needs to adjust to different environments. The following result gives the conditions for price-frame independence. It shows that the independence result holds only in special cases.

Proposition 8 *In the oligopoly model with $\alpha_2 < \alpha_1 = 1$,*
(i) for $n = 2$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$;
(ii) for $n \geq 3$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$ and $\alpha_2 = 0$, or for a particular non-uniformly random purchase rule $\{\phi_k\}_{k=1}^{n-1}$.

[32] If some firm charged a price lower than one with a positive probability, then at that price its demand would be positive (otherwise its equilibrium profit would be zero, which contradicts the fact that each firm earns $1/n$). But then consumer surplus would be positive. A contradiction.

Proof. At equilibrium, each firm's demand can be decomposed in two parts: the consumers who are insensitive to its price, and the consumers who are price-sensitive. Explicitly, we have

$$\pi(A,p)/p = \pi(A,1) + \{\lambda^{n-1} x_A(p)^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2 \phi_{n-k-1} + (1-\alpha_2)\phi_1\right]\} \text{ and}$$

$$\pi(B,p)/p = \pi(B,1) + \{(1-\alpha_2)(1-\lambda)^{n-1} x_B(p)^{n-1} + (1-\alpha_2)(1-\phi_1)\sum_{k=1}^{n-2} P_{n-1}^k x_B(p)^{n-k-1}\} .$$

Suppose $x_A(p) = x_B(p) = x(p)$, and the common support is $[p_0, 1]$. At equilibrium, $\pi(A,p) = \pi(B,p)$ must hold for any $p \in [p_0, 1]$.

(i) For $n = 2$, the last term in each demand function disappears. To have $\pi(A,p) = \pi(B,p)$ for any $p \in [p_0, 1]$, we need $\pi(A,1) = \pi(B,1)$, or equivalently $\dfrac{\lambda}{1-\lambda} = \dfrac{\phi_1 - \alpha_2/2}{1-\phi_1}$, and $\lambda = (1-\alpha_2)(1-\lambda)$, or equivalently $\dfrac{\lambda}{1-\lambda} = 1-\alpha_2$. It follows that these two conditions hold simultaneously if and only if $\phi_1 = 1/2$.

(ii) With $n \geq 3$, to have $\pi(A,p) = \pi(B,p)$ for any $p \in [p_0, 1]$, we need $\pi(A,1) = \pi(B,1)$ (see (17)), and

$$\lambda^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^{n-k-1} x(p)^{-k} \left[\alpha_2 \phi_k + (1-\alpha_2)\phi_1\right] = (1-\alpha_2)(1-\lambda)^{n-1} + (1-\alpha_2)(1-\phi_1)\sum_{k=1}^{n-2} P_{n-1}^k x(p)^{-k} .$$

(To derive the latter, we divided each side by $px(p)^{n-1}$ and relabelled k in $\pi(A,p)$ by $n-k-1$.) Then

$$\sum_{k=1}^{n-2} b_k x(p)^{-k} = (1-\alpha_2)(1-\lambda)^{n-1} - \lambda^{n-1} \tag{18}$$

where $b_k \equiv P_{n-1}^{n-k-1}\left[\alpha_2\phi_k + (1-\alpha_2)\phi_1\right] - P_{n-1}^k (1-\alpha_2)(1-\phi_1)$. Since the left-hand side of (18) is a polynomial of $1/x(p)$ and $x(p)$ is a decreasing function, (18) holds for all $p \in [p_0, 1]$ only if $b_k = 0$ for $k = 1, \cdots, n-2$ and the right-hand side is also zero. That is,

$$\left(\frac{\lambda}{1-\lambda}\right)^{n-1} = 1-\alpha_2 \text{ and} \tag{19}$$

$$\left(\frac{\lambda}{1-\lambda}\right)^{n-2k-1} = \frac{(1-\alpha_2)(1-\phi_1)}{\alpha_2\phi_k + (1-\alpha_2)\phi_1} \quad \text{for } k = 1, \cdots, n-2 . \tag{20}$$

If $\alpha_2 = 0$, both of them and (17) hold for $\phi_1 = 1/2$ (in which case, $\lambda = 1/2$). Beyond this special case, (20) pins down a decreasing sequence $\{\phi_k\}_{k=1}^{n-2}$ uniquely. Substituting (19) and (20) into (17), we can solve for ϕ_{n-1}. This means that, if $n \geq 3$ and $\alpha_2 > 0$, price-frame independence can hold only for a particular sequence of ϕ_k.[33] It is easy to verify that $\phi_k = 1/(1+k)$ does not satisfy these conditions. ■

[33] Note that, although $\{\phi_k\}_{k=1}^{n-2}$ solved from (20) is a decreasing sequence, still ϕ_{n-1}, which is solved from (17), may not be lower than ϕ_{n-2}. For example, when $n = 3$, one can check that

$$\phi_1 = \frac{1-\alpha_2}{2-\alpha_2} < \phi_2 = \frac{\phi_1 + 1/3 + \sqrt{1-\alpha_2}}{1+\sqrt{1-\alpha_2}},$$

which violates the requirement that ϕ_k is non-increasing in k.

Proof of Proposition 4: When frame B is also a simple frame (i.e., when $\alpha_2 = 0$), the equilibrium condition (17) for λ becomes $\frac{\lambda}{1-\lambda} = \left(\frac{\phi_1}{1-\phi_1}\right)^{1/(n-1)}$. It follows that λ tends to $1/2$ as $n \to \infty$.[34] Then industry profit $n\pi = n\phi_1 (1-\lambda)^{n-1}$ must converge to zero.[35]

Now consider $\alpha_2 > 0$. Since the left-hand side of (17) is bounded, it must be that $\lim_{n\to\infty} \lambda \leq 1/2$ (otherwise the right-hand side would tend to infinity). Since $\{\phi_k\}_{k=1}^{n-1}$ is a non-increasing sequence, the right-hand side of (17) is greater than

$$\frac{\alpha_2 (1-\phi_1)}{n} \sum_{k=1}^{n-2} C_{n-1}^k \left(\frac{\lambda}{1-\lambda}\right)^k = \frac{\alpha_2 (1-\phi_1)}{n} \left[\frac{1-\lambda^{n-1}}{(1-\lambda)^{n-1}} - 1\right] .$$

So it must be that $\lim_{n\to\infty} n (1-\lambda)^{n-1} > 0$, otherwise the right-hand side of (17) tends to infinity (given that $\lim_{n\to\infty} \lambda \leq 1/2$ and so $\lim_{n\to\infty}(1-\lambda^{n-1}) = 1$). This result implies that λ must converge to zero and industry profit $n\pi = n (1-\lambda)^{n-1} \left[\alpha_2 \phi_{n-1} + (1-\alpha_2)\phi_1\right]$ must be bounded away from zero as $n \to \infty$.

Proof of Proposition 5: Note that with the random purchase rule $\phi_k = 1/(1+k)$, (17) becomes

$$1 - \alpha_2 = 2\alpha_2 \sum_{k=1}^{n-2} \frac{C_{n-1}^k}{n-k+1} \left(\frac{\lambda}{1-\lambda}\right)^k + \left(\frac{\lambda}{1-\lambda}\right)^{n-1} , \tag{21}$$

and industry profit is

$$n\pi = n (1-\lambda)^{n-1} \left(\frac{\alpha_2}{n} + \frac{1-\alpha_2}{2}\right) . \tag{22}$$

(i) For $n = 2$, we have $\lambda = \frac{1-\alpha_2}{1+(1-\alpha_2)}$; and for $n = 3$, we have $\lambda = \frac{x}{1+x}$ with $x = \sqrt{4\alpha_2^2/9 + 1 - \alpha_2} - 2\alpha_2/3$. The latter is smaller if $x < 1 - \alpha_2$, which can be easily verified given that $\alpha_2 < 1$. The industry profit result follows from straightforward algebra calculation by using (22).

(ii) Consider the limit case $\alpha_2 \to 1$. The equilibrium condition (21) implies that λ should then tend to zero. As $\lambda \approx 0$, then $\lambda/(1-\lambda) \approx \lambda + \lambda^2$. For $n \geq 4$, the right-hand side of (21) can be approximated as

$$2\alpha_2[\frac{n-1}{n}(\lambda+\lambda^2) + \frac{n-2}{2}(\lambda+\lambda^2)^2] \tag{23}$$

by discarding all higher-order terms. (For $n = 3$, the right-hand side of (21) is approximately $\frac{4}{3}\alpha_2(\lambda+\lambda^2) + (\lambda+\lambda^2)^2$. One can check that the approximation result below still applies.)

Let $\alpha_2 = 1 - \varepsilon$ with $\varepsilon \approx 0$, and use the second-order (linear) approximation $\lambda \approx k_1\varepsilon + k_2\varepsilon^2$. Substituting them into (23) and discarding all terms of order higher than ε^2, we obtain

$$\frac{2(n-1)}{n} k_1 \varepsilon + \left(\frac{2(n-1)}{n}(k_2 - k_1) + \frac{n^2-2}{n} k_1^2\right) \varepsilon^2 .$$

[34] How λ varies with n also depends on the value of ϕ_1. If $\phi_1 > 1/2$, λ decreases to $1/2$ with n; if $\phi_1 = 1/2$, λ is a constant (equal to $1/2$); and if $\phi_1 < 1/2$, λ increases to $1/2$ with n.

[35] However, industry profit $n\pi$ can rise with n when n is small and ϕ_1 takes relatively extreme values. For example, when $\phi_1 = 0.95$ or 0.05, from $n = 2$ to 3, industry profit $n\pi$ increases from 0.095 to about 0.099.

Since the left-hand side of (21) is ε, we can solve

$$k_1 = \frac{n}{2(n-1)}; \quad k_2 = k_1 - \frac{n^2-2}{2(n-1)} k_1^2 .$$

As k_1 decreases with n, λ must decrease with n.

As $\varepsilon \approx 0$ (so that $\lambda \approx 0$), industry profit (for $n \geq 3$) can be approximated as

$$\begin{aligned} n\pi &= (1-\lambda)^{n-1}\left[1 + (\frac{n}{2} - 1)\varepsilon\right] \\ &\approx \left[1 - (n-1)\lambda + C_{n-1}^2 \lambda^2\right]\left[1 + (\frac{n}{2} - 1)\varepsilon\right] \\ &\approx 1 - \varepsilon + \frac{(n-2)n^2}{8(n-1)^2}\varepsilon^2 . \end{aligned}$$

Step two follows from discarding all terms of order higher than λ^2, and step three comes from substituting $\lambda \approx k_1\varepsilon + k_2\varepsilon^2$ and discarding all terms of order higher than ε^2. It is ready to see that the approximated industry profit increases with n. (Note that the first-order approximation of λ is not sufficient to tell how $n\pi$ varies with n.)

B.2 The case with $0 < \alpha_1 < \alpha_2 = 1$

We first show that there is no pure strategy equilibrium in this case either, and then prove Proposition 6.

Lemma 5 *In the oligopoly model with $0 < \alpha_1 < \alpha_2 = 1$, there is no equilibrium in which all firms use deterministic frames.*

Proof. We prove this lemma in three steps:

(a) In any pure strategy framing equilibrium, at most one firm uses frame A. Suppose to the contrary that at least two firms use frame A. Then, they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame B and a positive price. A contradiction.

(b) In any pure strategy framing equilibrium, at least one firm uses frame A. Suppose to the contrary that all firms use frame B. The only candidate equilibrium entails monopoly pricing $p = 1$ and each firm earns $1/n$. But then if one firm deviates to frame A and price $1 - \varepsilon$, it will earn $(1-\varepsilon)\left(\alpha_1\phi_{n-1} + 1 - \alpha_1\right)$. The reason is that, if the consumer is unable to compare prices in different frames (which happens with probability α_1), the deviator's demand is ϕ_{n-1}; if the consumer is able to compare prices in different frames (which happens with probability $1 - \alpha_1$), the deviator serves the whole market (because all other firms charge $p = 1$ and so are dominated options). As $\phi_{n-1} \geq 1/n$, the deviation profit is greater than $1/n$ for a sufficiently small ε and any $\alpha_1 \in (0, 1)$.

(c) The final possibility is that one firm uses A and all other firms use B. Suppose such an equilibrium exists. Let π_A be A firm's profit and π_B^j be the profit of a B firm indexed by j. (Notice that the B firms may use different pricing strategies and make different profits). Let p_A be the lowest price on which the A firm puts positive probability (it might be a deterministic

price). (i) Suppose that, at equilibrium, $\pi_A > \min\{\pi_B^j\}$. Then, if the B firm which earns the least deviates to frame A and a price $p_A - \varepsilon$, it will replace the original A firm and have a demand at least equal to the original A firm's demand since it now charges a lower price *and* faces fewer competitors.[36] So, this deviation is profitable at least when ε is close to zero. A contradiction. (ii) Suppose now that, at equilibrium, $\pi_A \leq \min\{\pi_B^j\}$. Notice that $\pi_A \geq 1/n$, otherwise the A firm would deviate to frame B and a price $p = 1$, and make profit $1/n$. As industry profit cannot exceed one, all firms must earn $1/n$ at the candidate equilibrium and consumer surplus is zero. This also implies that all firms must be charging the monopoly price. But then any B firm has an incentive to deviate to frame A and price $1 - \varepsilon$, in which case it makes profit $(1-\varepsilon)\left(\alpha_1\phi_{n-2} + 1 - \alpha_1\right) > 1/n$ for a sufficiently small ε. A contradiction. ■

We now characterize a symmetric mixed-strategy equilibrium (λ, F_A, F_B) where λ is the probability of using frame A, F_A is defined on $S_A = [p_0^A, 1)$ and is atomless, and F_B is degenerate on $S_B = \{1\}$.

Along the equilibrium path, if firm i uses frame A and charges $p \in [p_0^A, 1)$, its profit is given by

$$\pi(A,p) = p\sum_{k=0}^{n-1} P_{n-1}^k x_A(p)^k \left(\alpha_1\phi_{n-k-1} + 1 - \alpha_1\right) . \tag{24}$$

This expression follows from the fact that, when k other firms also use frame A, firm i has a positive demand only if all other A firms charge prices higher than p. Conditional on that, with probability α_1, the consumer is confused by frame differentiation and buys from firm i with probability ϕ_{n-k-1} (since all $n-k-1$ firms which use B are undominated); with probability $1-\alpha_1$, the consumer can compare A and B and, because all B firms charge price $p_B = 1 > p$ and consequently are dominated, she only buys from firm i.

A firm's equilibrium profit is equal to

$$\pi = \lim_{p\to 1} \pi(A,p) = (1-\lambda)^{n-1}\left(\alpha_1\phi_{n-1} + 1 - \alpha_1\right) . \tag{25}$$

Then the expression for $F_A(p)$ follows from $\pi(A,p) = \pi$, and p_0^A satisfies $\pi(A, p_0^A) = \pi$. Both of them are well defined.

If firm i uses B and charges $p = 1$, then its profit is

$$\pi(B,1) = \frac{(1-\lambda)^{n-1}}{n} + \alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k \frac{1-\phi_{n-k}}{n-k} . \tag{26}$$

Notice that firm i has a positive demand only if all other firms also use frame B, or there are A firms but the consumer is unable to compare prices in different frames.

[36] When the consumer is unable to compare prices in different frames, the deviator's demand is ϕ_{n-2} which is (weakly) greater than ϕ_{n-1}, the original A firm's demand in this case. When the consumer is able to compare prices in different frames, the deviator is more likely to dominate the remaining B firms (and so to have a higher expected demand) than the original A firm.

The equilibrium condition $\pi(B,1) = \lim_{p\to 1}\pi(A,p)$ pins down a well-defined λ:

$$\frac{1-1/n}{\alpha_1} + \phi_{n-1} - 1 = \sum_{k=1}^{n-1} \frac{C_{n-1}^k\left(1-\phi_{n-k}\right)}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^k . \tag{27}$$

The left-hand side of (27) is positive given that $\phi_{n-1} \geq 1/n$, and the right-hand side is increasing in λ from zero to infinity. Hence, for any given $n \geq 2$ and $\alpha_1 \in (0,1)$, equation (27) has a unique solution λ in $(0,1)$.

To complete the proof of Proposition 6, we only need to rule out profitable deviations from the proposed equilibrium. First, consider two possible deviations with frame A: (i) a deviation to $\left(A, p < p_0^A\right)$ is not profitable as the firm does not gain market share, but loses on prices; (ii) a deviation $(A, p = 1)$ is not profitable either, since the deviator's profit is $(1-\lambda)^{n-1}\phi_{n-1} < \pi$.

Let us now consider a deviation to $\left(B, p \in [p_0^A, 1)\right)$. Deviator's profit is

$$\hat{\pi}(B,p) = p\pi(B,1) + p(1-\alpha_1)\sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k .$$

This expression captures the fact that when $n-1$ other firms also use B, or when $k \geq 1$ firms use A and the consumer is confused between A and B, firm i's demand does not depend on its price so that it is equal to $\pi(B,1)$. When $k \geq 1$ firms use A and the consumer is not confused between A and B, all other B firms (which charge price $p = 1$) are dominated by the cheapest A firm, and the consumer buys from firm i only if the cheapest A firm charges a price greater than p. Notice that, from $\pi(A,p) = \pi$ for $p \in [p_0^A, 1)$, the second term in $\hat{\pi}(B,p)$ is equal to

$$\pi - p\pi - p\alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k \phi_{n-k-1} .$$

Then, $\hat{\pi}(B,p) < p\pi + \pi - p\pi = \pi$. The deviation to $\left(B, p < p_0^A\right)$ will result in a lower profit. This completes the proof.

Proof of Proposition 7:

From (27), it follows that $\lambda \to 1$ as $\alpha_1 \to 0$. Let $\alpha_1 = \varepsilon$ with $\varepsilon \approx 0$, and $\lambda = 1-\delta$ with $\delta \approx 0$. Then the right-hand side of (27) can be approximated as

$$(1-\phi_1)\left(\frac{1-\delta}{\delta}\right)^{n-1} \approx \frac{1-\phi_1}{\delta^{n-1}} ,$$

since only the term with $k = n-1$ matters when $\delta \approx 0$. Hence, from (27), we can solve

$$\delta \approx \left(\frac{1-\phi_1}{\frac{1}{\varepsilon}(1-\frac{1}{n}) + \phi_{n-1} - 1}\right)^{1/(n-1)} \approx \left(\frac{n(1-\phi_1)\varepsilon}{n-1}\right)^{1/(n-1)} .$$

The second step follows from the fact that $\phi_{n-1} - 1$ is negligible compared to $\frac{1}{\varepsilon}(1-\frac{1}{n})$. Given that $\varepsilon \approx 0$, it is not difficult to see that δ increases with n (e.g., one can show that $\ln\delta$ increases with n). Hence, λ decreases with n. As $\varepsilon \approx 0$, industry profit is

$$n\pi = n\delta^{n-1}[1 + (\phi_{n-1} - 1)\varepsilon] \approx \frac{n^2(1-\phi_1)\varepsilon}{n-1}$$

by discarding the term of ε^2. Clearly, $n\pi$ increases with n.

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