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Abstract

We analyze a mixed oligopoly with free entry by private firms. It is assumed that a state-owned enterprise (SOE) maximizes an increasing function of output, subject to a break-even constraint. We first show that, because of instability, the industry cannot contain more than one SOE. Then we establish an irrelevance result: if the SOE's cost disadvantage relative to private firms is not too large, then aggregate output, aggregate costs and welfare are the same with and without the SOE. However, for this range of cost disadvantage an SOE monopoly yields higher welfare. Implications for privatization policy are suggested.

JEL Classification: H32, L32, P23

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Mixed Oligopoly and Entry

1 Introduction

In the extensive literature on mixed oligopoly it is commonly assumed that a state-owned enterprise (SOE) behaves strategically to maximize either social welfare or a weighted objective function in which welfare is one of the arguments, with the budget constraint rarely being considered.¹ A different approach was taken in some early contributions. Crémer, Marchand and Thisse (1989) assume the SOE is welfare-maximizing, but subject to a break-even constraint, and they note that their results would still hold for what might be deemed a more realistic ‘bureaucratic’ objective such as output maximization subject to this constraint. The latter case is argued by Estrin and de Meza (1995) to reflect the relevant descriptive literature, and they explore it in detail. However, the implications of incorporating free entry by private firms into this framework have not been fully explored. Free entry is empirically relevant, given the trend to deregulation and privatization in industries in which SOEs operate (see Florio and Fecher, 2011) and, as we shall see, theoretically, it throws up some intriguing results.

We assume that an SOE maximizes a function that is increasing in its own output, subject to a break-even constraint. This maximand is general enough to cover a wide range of managerial specifications, including maximization of total

¹The early literature is surveyed by De Fraja and Delbono (1990). For extensive references to more recent literature, see Tomaru, Nakamura and Saito (2011). In some contributions it has been assumed that an SOE maximizes a linear combination of welfare and profits, either because it are partly private-owned owned (Matsumura, 1998), or as a result of an incentive contract strategically-imposed by the government (Barros, 1995). Also, there is also a literature in which the SOE’s objective is a function of the bargaining weights of the different factions in the firm (De Donder and Roemer, 2009).

revenue, output, employment, and the rate of growth of output. The assumption that there is a break-even constraint is a normalization in the sense that, within limits discussed at the end of the paper, the argument is unaffected if the more general assumption of a profit constraint (of either sign) is made. Compared to the objective used in most mixed oligopoly literature, this formulation is informationally much less demanding to enforce, allowing decentralized decision-making.

As a prelude to examining mixed oligopoly, we consider an industry composed of at least two identical SOEs, but with no private firms, and we show that a symmetric stable equilibrium does not exist. As well as providing a rationale for the assumption we maintain in the rest of the paper, that only one SOE need be considered in a mixed oligopoly, this suggests a novel interpretation of the need for planning in industries with more than one SOE.

We then consider mixed oligopoly on the assumption that an SOE may have a cost disadvantage relative to private firms. Perhaps surprisingly, we show that if the cost disadvantage is not too large, the presence of a high-cost SOE is immaterial for aggregate output and costs, and therefore for welfare. It follows that there would be no welfare gain from privatization of the SOE (or indeed from nationalizing private firms).² Moreover, for the same range of cost parameters a monopoly SOE would yield a higher level of welfare than either of the alternatives, but we suggest that a monopoly SOE may have drawbacks in dynamic terms. However, if the cost disadvantage of the SOE is relatively large, a mixed oligopoly is not viable, while an all-private oligopoly outperforms a monopoly SOE in welfare terms.

The model is set up in Section 2. The state-owned enterprise equilibrium is

²As in much of the mixed oligopoly literature, we assume that a firm's cost parameters depend only on its ownership.

examined in Section 3, and our main results, for mixed oligopoly, are developed in Section 4. Section 5 concludes.

2 The Model

We consider a homogeneous-good oligopoly facing a linear demand function,

$$P(Q) = a - Q, \quad (1)$$

where $P(\cdot)$ is unit price and Q total output. The industry is populated by set S of m identical SOEs (each indexed s) and set P of n identical private firms (each indexed p), with all firms entertaining Cournot conjectures. The output of an SOE is q^s and that of a private firm is q^p , and so $Q = mq^s + nq^p$. The total cost function for any firm i is linear:

$$C(q^i) = c^i q^i + k^i, \quad c^i \geq 0; k^i \geq 0, \quad i = s, p. \quad (2)$$

An SOE chooses output q^s to maximize a function G that is increasing in q^s , subject to a break-even constraint. Thus, it solves the programme,

$$\max_{q^s} G(q^s, \sum_{\substack{\bar{s} \in S \\ \bar{s} \neq s}} q^{\bar{s}}, \sum_{p \in P} q^p) \text{ subject to } \pi^s \geq 0 \quad (3)$$

where

$$\pi^s \equiv [a - c^s - q^s - \sum_{\substack{\bar{s} \in S, \\ \bar{s} \neq s}} q^{\bar{s}} - \sum_{p \in P} q^p] q^s - k^s. \quad (4)$$

The only restrictions that we impose on $G(\cdot)$ are that it is well-behaved, that the

second-order conditions are satisfied, and that in equilibrium $G_{q^s} > 0$. Denoting by λ and $L(\cdot)$ the Lagrange multiplier and Lagrangian, respectively, we obtain

$$L_{q^s} = G_{q^s} + \lambda\pi_{q^s}^s = 0 \quad (5)$$

$$L_\lambda = \pi^s = 0 \quad (6)$$

Given that $G_{q^s} > 0$, the break-even constraint binds, and it follows that the SOE produces past the profit-maximizing output (i.e., in equilibrium, $\pi_{q^s}^s < 0$).

Note that the first-order condition (5) plays no role in determining the equilibrium values of the programme. Rather, it is the zero-profit condition (6) that does all the work. To ensure that an SOE may be able to satisfy its break-even constraint, we assume that it would be able to make non-negative profit as a monopolist in the industry, i.e.,

$$a \geq c^s + 2\sqrt{k^s}. \quad (7)$$

As is common in the mixed oligopoly literature, and is substantiated by a large body of empirical evidence (see, e.g., Roland, 2008), we shall assume that an SOE does not have a cost advantage, and may have a cost disadvantage, relative to a private firm. For simplicity, we shall examine separately the case in which the SOE has a marginal cost disadvantage from the case in which the SOE has higher fixed costs. Specifically, we consider two cases: (i) either $c^s > c^p = 0$ (where, without loss of generality, the profit-maximizers' marginal cost is normalized to zero) and $k^s = k^p$; or $c^s = c^p = 0$ and $k^s > k^p$.³

³We exclude the possibility that $c^s > c^p$ while $k^s < k^p$, or that $c^s < c^p$ while $k^s > k^p$. Our analysis is easily extended to these cases, but the algebra is made more complicated without

3 State-Owned Enterprise Equilibrium

In this section we set $n = 0$ and examine the conditions under which a state-owned industry can sustain more than a single SOE. We highlight a previously unnoticed feature of such markets in the following:

Proposition 1 *For the functional forms (1)-(2), whenever identical firms choose output levels simultaneously to maximize an objective function increasing in their own output, subject to a break-even constraint, the resulting symmetric equilibrium is unstable.*

Proof: see appendix.

The proposition is illustrated in Figure 1 for $m = 2$. The SOEs' best-response functions coincide with their respective zero-profit conditions. For each firm i , there are two segments of these best-response functions, which are marked $\pi^i(q_1, q_2) = 0$ and $\pi^{i0}(q_1, q_2) = 0$ ($i = 1, 2$). On the $\pi^i(\cdot)$ -segment firm i produces a positive output, while on the π^{i0} -segment it does not produce. The symmetric equilibrium E is on the positive-output segments, but is unstable, as firm 2's best response curve cuts firm 1's best-response curve from above. This instability result cannot be ascribed to our maintained assumptions (linear demand and costs, and identical firms) as, if the firms were profit-maximizers, their best-response functions would be shown by the curves labelled $B^1(q^1, q^2)$ and $B^2(q^1, q^2)$, and a unique stable symmetric equilibrium at E'' would obtain. Note that, with two SOEs there are also stable asymmetric equilibria at A and H , but these require that one firm be inactive.

changing the results qualitatively.

[Figure 1]

The inherent instability of an industry made of two or more SOEs may be interpreted as an indication of the incompatibility between state ownership of firms and decentralized decision-making. Proposition 1 can be read as implying that an industry with multiple SOEs can only operate in a market if there is *quantity guidance*. This could be achieved through direct command from the centre, or it might be the result of exogenous constraints on output, as was common in the early years of transition economies (Blanchard and Kremer, 1997). This is also illustrated in Figure 1: firm i has a binding output constraint $q^i \leq \bar{q}^i$ ($i = 1, 2$). Then its best-response curve becomes $q^i - \bar{q}^i \leq 0$ and the resulting symmetric equilibrium E' is stable.

4 Mixed Oligopoly

In view of Proposition 1, we henceforth assume that there is only one SOE. If the cost parameters are such that both the SOE and at least one private profit-maximizing firm produce positive amounts in equilibrium, we call this a *proper mixed oligopoly*. We establish an irrelevance result: given free private entry, a proper mixed oligopoly and a purely private oligopoly result in the same levels of aggregate output and aggregate costs, and therefore of welfare. We then show, however, for the same range of cost parameters, that a monopoly SOE would generate a higher level of welfare, while if the SOE has too great a cost disadvantage for a proper mixed oligopoly to be viable, welfare is greater with an all-private oligopoly than with a monopoly SOE.

The SOE's programme is still defined by (3)-(4), but with $m = 1$. The number

n of identical private firms is determined by a zero-profit free entry condition.

Each private firm solves the following programme:

$$\max_{q^p} \pi^p(q^p) = \max_{q^p} \left[(a - q^s - \sum_{\substack{\bar{p} \in P \\ \bar{p} \neq p}} q^{\bar{p}} - q^p) q^p - k^p \right], \quad (8)$$

given the free-entry condition,

$$[a - q^s - \sum_{\substack{\bar{p} \in P \\ \bar{p} \neq p}} q^{\bar{p}} - q^p] q^p = k^p. \quad (9)$$

From the first-order conditions for maximization of (8), together with (6) and (9), if there is an interior solution, it is unique (we have a proper mixed oligopoly equilibrium). Denoting interior solution values by an asterix,

$$q^{s*} = \frac{k^s}{\sqrt{k^p} - c^s}; \quad q^{p*} = \sqrt{k^p}; \quad n^* = \frac{a}{\sqrt{k^p}} - \frac{k^s}{(\sqrt{k^p} - c^s) \sqrt{k^p}} - 1. \quad (10)$$

The condition for this equilibrium to obtain with at least one private firm may be written in terms of a non-empty interval for c^s (or it may be turned around to be a non-empty interval for k^s).

Lemma 1 *A proper mixed oligopoly equilibrium exists if $c^s \in [0, \sqrt{k^p} - k^s / (a - 2\sqrt{k^p})]$.*

The upper bound for c^s is given by setting $n^* \geq 1$ in (10), while the lower bound is the condition that $c^s \geq c^p = 0$. (10) yields the comparative statics of

marginal and fixed costs:

$$\frac{\partial n^*}{\partial c^s} = -\frac{k^s}{\left(\sqrt{k^p} - c^s\right)^2 \sqrt{k^p}} < 0; \quad \frac{\partial n^*}{\partial k^s} = -\frac{1}{\left(\sqrt{k^p} - c^s\right) \sqrt{k^p}} < 0. \quad (11)$$

An increase in either of the SOE's cost disadvantages *reduces* entry. Higher cost parameters for the SOE leads it to produce more in order to satisfy its break-even constraint, and the negative impact on price causes there to be less private entry.

By way of comparison, in an all-private oligopoly with free entry (10) still holds, but with the SOE-terms deleted; i.e., denoting values in this solution with a circumflex,

$$\hat{q}^p = \sqrt{k^p}; \quad \hat{n} = \frac{a}{\sqrt{k^p}} - 1. \quad (12)$$

From (10) and (12), $q^{s*} + n^* q^{p*} = \hat{n} \hat{q}^p = a - \sqrt{k^p}$ and $c^s q^{s*} + k^s + n^* k^f = \hat{n} k^p = a - \sqrt{k^p}$. Thus we have the following 'irrelevance' result:

Proposition 2 *For the functional forms (1)-(2), aggregate output, aggregate costs and net social welfare are the same in a proper mixed oligopoly as in an all-private oligopoly.*

Despite the difference in objective functions between the SOE and private firms, aggregate output is the same in the two equilibria.⁴ There are two contradictory forces behind this result. On the one hand, the SOE's higher costs give it a competitive disadvantage relative to private firms. On the other, the SOE's maximization of a function that is increasing in its level of output acts as a credible commitment

⁴Estrin and de Meza (1995, Proposition 5) argue, by verbal reasoning, that (using our notation) $n^* + 1 = \hat{n}$. However, from (10) and (12), for this to be true it is necessary that $k^s = \left(\sqrt{k^p} - c^s\right) \sqrt{k^p}$. If, say, $k^s = k^p$, this only holds if $c^s = 0$, there being no difference in the cost parameters of SOEs and private firms.

to producing past its profit-maximizing point, and this gives the SOE a competitive advantage. In equilibrium, these two forces balance out, with aggregate output determined by profits being driven down to zero irrespective of whether this is the result of free entry or a binding zero-profit constraint. Notice that a profit-maximizer produces the same amount of output in a proper mixed oligopoly as in an all-private oligopoly, with free entry in both cases determining the total number of private firms. The powerful effect of free entry makes the difference in objective functions irrelevant.

Furthermore, despite the cost-function differences between the SOE and private firms, the irrelevance result also holds with respect to aggregate costs, and the immediate corollary is that it holds for net social welfare. An implication is that, at least for the canonical form sketched above, if a proper mixed oligopoly is viable there is no rationale for either *partial nationalization* (i.e., switching from an all-private oligopoly to a mixed oligopoly) or *full privatization* (i.e., switching from a mixed oligopoly to all-private oligopoly).

However, another policy option for the government might be to prohibit private production altogether (i.e., *full nationalization*). Our final proposition considers the implications for social welfare.

Proposition 3 *For the functional forms (1)-(2), with free private entry (a) if $c^s \in [0, \sqrt{k^p} - k^s/(a - 2\sqrt{k^p})]$ an SOE monopoly is welfare-superior to a proper mixed oligopoly or all-private oligopoly; (b) if $c^s \in (\sqrt{k^p} - k^s/(a - 2\sqrt{k^p}), a - 2\sqrt{k^s}]$ all-private oligopoly is welfare-superior to a monopoly SOE.*

Proof: see appendix.

We saw previously that for the cost range in which a proper mixed oligopoly

is viable, the existence of an SOE does not improve welfare, for the proper mixed oligopoly merely replicates the outcome that would obtain with an all-private oligopoly. Proposition 3 goes further, specifying that, for this cost range, a monopoly SOE would generate greater welfare. Thus, for the cost range considered, it is not SOE production that causes welfare to be forgone; rather, it is SOE production together with private production that causes the problem.⁵ However, for the higher cost range in which a monopoly SOE is viable, but a proper mixed oligopoly is not, welfare is greater with free-entry all-private oligopoly than with a monopoly SOE. The higher cost level for the SOE rules it out as an instrument for enhancing social welfare.⁶

The break-even constraint for the SOE plays an important role in our analysis, although within limits it is essentially a normalization. We end by discussing this constraint further. Suppose that, generalizing, the SOE is required to make a profit of at least π_0^s , which may take either sign. Then, in (4), we must replace fixed cost k^s the quasi-fixed cost $k^s + \pi_0^s \equiv k_0^s$. (Variable profit must cover both the fixed cost and the profit requirement.) Our qualitative results still hold if $k_0^s > k^p$, provided k_0^s is less than the level at which the SOE's output would be at the profit-maximizing level, although the range of parameters $\{a, c^s, k^s, k^p\}$ that allows the existence of a proper mixed oligopoly must be amended. However, if π_0^s is sufficiently negative, $k_0^s \leq k^p$. Then, if $c^s = 0$ there would be no private entry: the competitive advantage for the SOE from being allowed to make losses

⁵In the literature on mixed oligopoly in which it is assumed that the SOE is welfare-maximizing, there being no free entry, it is found that if there is no cost disadvantage for the SOE, nationalization of all private firms is optimal (De Fraja and Delbono, 1990). With our differing assumptions we specify the extent of cost-disadvantage for which nationalization is optimal.

⁶As we have already noted, if c^s is higher than the ranges specified in Proposition 3 (i.e., if $c^s > a - 2\sqrt{k^s}$) an SOE is not viable, being unable to meet its break-even constraint.

of this size makes private production unprofitable. If $c^s > 0$, so that the SOE has a marginal cost disadvantage along with a quasi-fixed cost advantage, we are in the case mentioned in n. 2.

5 Conclusion

With a ‘managerial’ model of an SOE, we have examined the equilibrium for a mixed oligopoly with free entry by private firms. After first ruling out multiplicity of SOEs because of instability, we have established an irrelevance result: provided the (single) SOE does not have too large a cost disadvantage, mixed oligopoly generates the same solution, in terms of aggregate costs, aggregate output and welfare, as an all-private oligopoly. But we have also shown that for this range of cost disadvantage, a monopoly SOE yields a higher level of welfare. Nonetheless, this does not necessarily mean that mixed oligopoly should be ruled out on welfare grounds. For we have abstracted from the potential endogeneity of the cost function. Thus, for example, in practice, in the absence of competition, the costs of the SOE might drift up into the range in which all-private oligopoly outperforms each of the alternatives in welfare terms.

Appendix: Proofs

Proposition 1

From Vives (2001, p.102) a necessary condition for stability is that for any pair of firms i and j ,

$$\left| \left(\frac{dq^j}{dq^i} \right) \pi^i = 0 \right| = \frac{\partial \pi^i / \partial q^i}{\partial \pi^i / \partial q^j} > \left| \left(\frac{dq^j}{dq^i} \right) \pi^j = 0 \right| = \frac{\partial \pi^j / \partial q^i}{\partial \pi^j / \partial q^j} \quad (\text{A1})$$

Differentiating (6) totally and using (4),

$$\left(\frac{dq^j}{dq^i}\right) \pi^i = 0 = -\frac{\partial\pi^i/\partial q^i}{\partial\pi^i/\partial q^j} = -\frac{1}{q^i}(a - 2q^i - q^j - c^i), \quad i \neq j = 1, 2. \quad (\text{A2})$$

In the solution $\mu^i > 0$ and so, since $\partial G/\partial q^i > 0$, we have from (6) that $\partial\pi^i/\partial q^i < 0$. Therefore, from (4), $\partial\pi^i/\partial q^i = a - 2q^i - q^j - c^i < 0$; also $\partial\pi^i/\partial q^j = -q^i$. But $a - q^i - q^j - c^i$ is the markup per unit of output over variable cost for firm i , which, to cover fixed cost k^i , must be positive. Therefore $-q^i < a - 2q^i - q^j - c^i < 0$, and so the expression in (A2) takes a value in the range $(0, 1)$. Performing the same calculation with the roles of i and j reversed, we have that $-(\partial\pi^j/\partial q^j)/(\partial\pi^j/\partial q^i) \in (0, 1)$. Since the last expression in (A1) equals $1/[-(\partial\pi^j/\partial q^j)/(\partial\pi^j/\partial q^i)]$, its value is greater than unity. Therefore (A1) must be violated.

Proposition 3

We examine aggregate output since this is a sufficient statistic for net social welfare. From (4), the output of a monopoly SOE is $[a - c^s + \sqrt{(a - c^s)^2 - 4k^s}]/2 \equiv \bar{q}^s$. From (12), the aggregate output of a proper mixed oligopoly is $q^{s*} + n^*q^{p*} = a - \sqrt{k^p}$. After some manipulation, it is found that

$$[a - c^s + \sqrt{(a - c^s)^2 - 4k^s}]/2 \gtrless a - \sqrt{k^p} \text{ as } c^s \lesseqgtr \sqrt{k^p} - k^s/(a - \sqrt{k^p}). \quad (\text{A3})$$

But, from Lemma 1, a proper mixed oligopoly is only viable for $c^s \leq \sqrt{k^p} - k^s/(a - 2\sqrt{k^p})$. Since $\sqrt{k^p} - k^s/(a - \sqrt{k^p}) < \sqrt{k^p} - k^s/(a - 2\sqrt{k^p})$, it follows from (A3) that, for $c^s \leq \sqrt{k^p} - k^s/(a - 2\sqrt{k^p})$, $\bar{q}^s > q^{s*} + n^*q^{p*}$. Therefore a monopoly SOE social yields greater social welfare than a proper mixed oligopoly (or an all-private oligopoly (given Proposition 2)).

For $c^s \in (\sqrt{k^p} - k^s/(a - 2\sqrt{k^p}), a - 2\sqrt{k^s}]$ a proper mixed oligopoly cannot obtain, but a SOE monopoly is viable, while, for an all-private oligopoly, aggregate output $\hat{n}\hat{q}^p$ is still $a - \sqrt{k^p}$. A parallel argument applies to that given above. Using (A3) and the inequality $\sqrt{k^p} - k^s/(a - \sqrt{k^p}) < \sqrt{k^p} - k^s/(a - 2\sqrt{k^p})$, it is found that $\bar{q}^s < \hat{n}\hat{q}^p$, social welfare being higher for the all-private oligopoly than the SOE monopoly.

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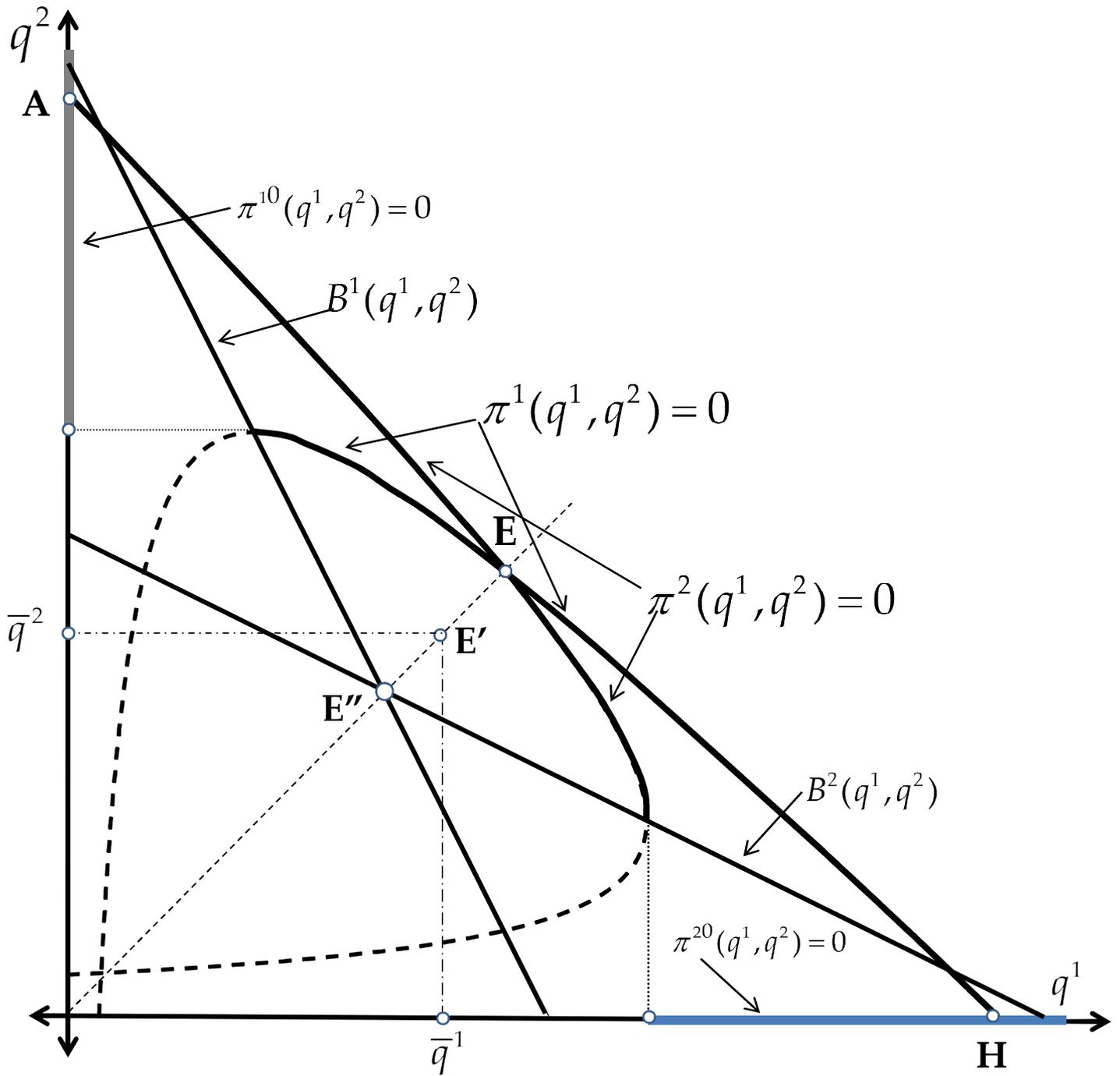


Figure 1: SOE duopoly